Effective Measurements for Structural Dynamics Testing

PART I

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Digital Fourier analyzers have opened a new era in structural dynamics testing. The ability of these systems to quickly and accurately measure a set of structural frequency response functions and then operate on them to extract modal parameters is having a significant impact on the product design and development cycle. Part I of this article is intended to introduce the structural dynamic model and the representation of modal parameters in the Laplace domain. The concluding section explains the theory for measuring structural transfer functions with a digital analyzer. Part II will be directed at presenting various practical techniques for measuring these functions with sinesoidal, transient and random excitation. New advances in random excitation will be presented and digital techniques for separating closely coupled modes via increased frequency resolution will be introduced.

Structural Dynamics and Modal Analysis

Understanding the dynamic behavior of structures and structural components is becoming an increasingly important part of the design process for any mechanical system. Economic and environmental considerations have advanced to the state where over-design and less than optimum performance and reliability are not readily tolerated. Customers are demanding products that cost less, last longer, are less expensive to operate, while at the same time they must carry more pay-load, run quieter, vibrate less, and fail less frequently. These demands for improved product performance have caused many industries to turn to advanced structural dynamics testing technology.

The use of experimental structural dynamics as an integral part of the product development cycle has varied widely in different industries. Aerospace programs were among the first to apply these techniques for predicting the dynamic performance of fight vehicles. This type of effort was essential because of the weight, safety, and performance constraints inherent in aerospace vehicles. Recently, increased consumer demand for fuel economy, reliability, and superior vehicle ride and handling qualities have been instrumental in making structural dynamics testing an integral part of the automotive design cycle. An excellent example was reported in the cover story article on the new Cadillac Seville from Automotive Industries, April 15, 1975.

"The most radical use of computer technology which 'will revolutionize the industry' . . . is dynamic structural analysis, or Fourier analysis as it is commonly known. It was this technique, in conjunction with others, that enabled Cadillac to 'save a mountain of time and money,' and pare down the number of prototypes necessary. It also did away with much trial and error on the solution of noise and vibration problems."

In order to understand the dynamic behavior of a vibrating structure, measurements of the dynamic properties of the structure and its components are essential. Even though the dynamic properties of certain components can be determined with finite computer techniques, experimental verification of these results are still necessary in most cases.

One area of structural dynamics testing is referred to as modal analysis. Simply stated, *modal analysis* is the process of characterizing the dynamic properties of an elastic structure by identifying its modes of vibration. That is, each mode has a specific natural frequency and damping factor which can be identified from practically any point on the structure. In addition, it has a characteristic "mode shape" which defines the mode spatially over the entire structure.

Once the dynamic properties of an elastic structure have been characterized, the behavior of the structure in its operating environment can be predicted and, therefore, controlled and optimized.

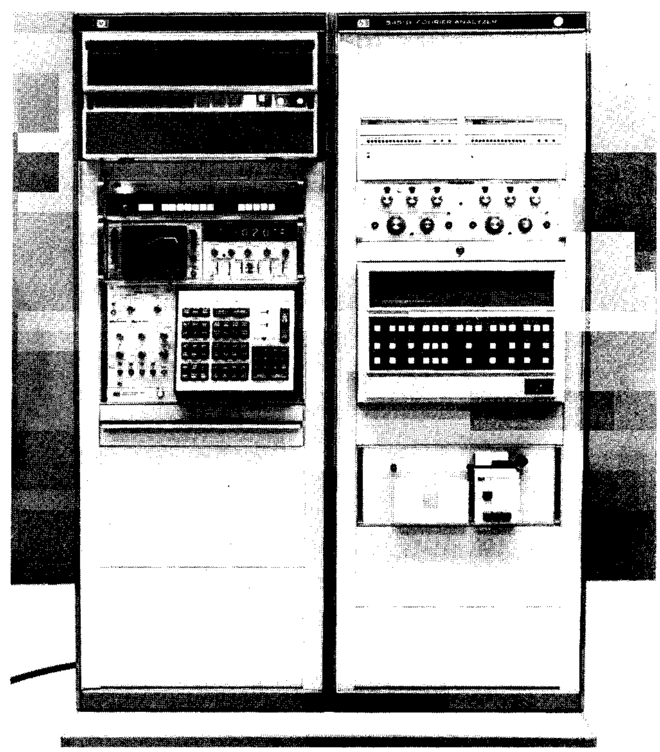

Figure 1—The HP 5451 B Fourier Analyzer is typical of modern digital analyzers that can be used for acquisition and processing of structural dynamics data.

In general, modal analysis is valuable for three reasons:

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1) Modal analysis allows the verification and adjusting of the mathematical models of the structure. The equations of motion are based on an idealized model and are used to predict and simulate dynamic performance of the structure. They also allow the designer to examine the effects of changes in the mass, stiffness, and damping properties of the structure in greater detail. For anything except the simplest structures, modeling is a formidable task. Experimental measurements on the actual hardware result in a physical check of the accuracy of the mathematical model. If the model predicts the same behavior that is actually measured, it is reasonable to extend the use of the model for simulation, thus reducing the expense of building hardware and testing each different configuration. This type of modeling plays a key role in the design and testing of aerospace vehicles and automobiles, to name only two.
2) Modal analysis is also used to locate structural weak points. It provides added insight into the most effective product design for avoiding failure. This often eliminates the tedious trial and error procedures that arise from trying to apply inappropriate static analysis techniques to dynamic problems.
3) Modal analysis provides information that is essential in eliminating unwanted noise or vibration. By understanding how a structure deforms at each of its resonant frequencies, judgments can be made as to what the source of the disturbance is, what its propagation path is, and how it is radiated into the environment.

In recent years, the advent of high performance, low cost minicomputers, and computing techniques such as the fast Fourier transform have given birth to powerful new "instruments" known as digital Fourier analyzers (see Figure 1). The ability of these machines to quickly and accurately provide the frequency spectrum of a time-domain signal has opened a new era in structural dynamics testing. It is now relatively simple to obtain fast, accurate, and complete measurements of the dynamic behavior of mechanical structures, via transfer function measurements and modal analysis.

Techniques have been developed which now allow the modes of vibration of an elastic structure to be identified from measured transfer function data,[1,2]. Once a set of transfer (frequency response) functions relating points of interest on the structure have been measured and stored, they may be operated on to obtain the modal parameters; i.e., the natural frequency, damping factor, and characteristic mode shape for the predominant modes of vibration of the structure. Most importantly, the modal responses of many modes can be measured simultaneously and complex mode shapes can be directly identified, permitting one to avoid attempting to isolate the response of one mode at a time, i.e., the so called "normal mode" testing concept.

The purpose of this article is to address the problem of making effective structural transfer function measurements for modal analysis. First, the concept of a transfer function will be explored. Simple examples of one and two degree of freedom models will be used to explain the representation of a mode in the Laplace domain. This representation is the key to understanding the basis for extracting modal parameters from measured data. Next, the digital computation of the transfer function will be shown. In Part II, the advantages and disadvantages of various excitation types and a comparison of results will illustrate the importance of choosing the proper type of excitation. In addition, the solution for the problem of inadequate frequency resolution, non-linearities and distortion will be presented.

The Structural Dynamics Model

The use of digital Fourier analyzers for identifying the modal properties of elastic structures is based on accurately measuring structural transfer (frequency response) functions. This measured data contains all of the information necessary for obtaining the modal (Laplace) parameters which completely define the structures' modes of vibration. Simple one and two degree of freedom lumped models are effective tools for introducing the concepts of a transfer function, the *s*-plane representation of a mode, and the corresponding modal parameters.

The idealized single degree of freedom model of a simple vibrating system is shown in Figure 2. It consists of a spring, a damper, and a single mass which is constrained to move along one axis only. If the system behaves linearly and the mass is subjected to any arbitrary time varying force, a corresponding time varying motion, which can be described by a linear second order ordinary differential equation, will result. As this motion takes place, forces are generated by the spring and damper as shown in Figure 2.

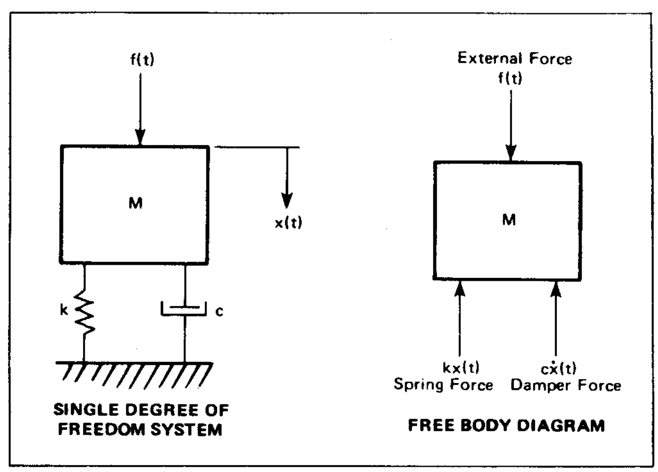


Figure 2—Idealized single degree of freedom model.

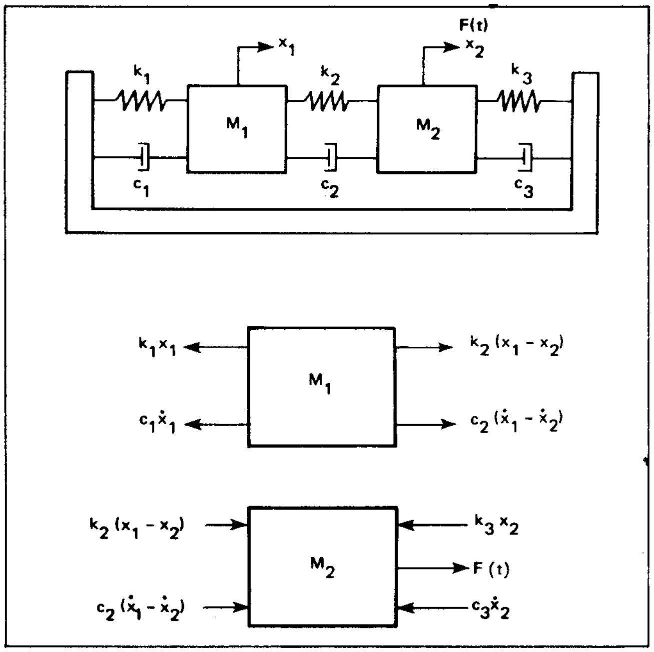


Figure 3—A two degree of freedom model.

The equation of motion of the mass m is found by writing Newton's second law for the mass ($\sum \mathbf{F}_{ext} = ma$), where *ma* is a real inertial force,

$$f(t) - kx(t) - c\dot{x}(t) = m\ddot{x}(t) \tag{1}$$

where $\dot{x}(t)$ and $\ddot{x}(t)$ denote the first and second time derivatives of the displacement x(t). Rewriting equation (1) results in the more familiar form:

$$m\ddot{x} + c\dot{x} + kx = f(t) \tag{2}$$

where: *f*(*t*) = applied force
x = resultant displacement
$\dot{x}$ = resultant velocity
$\ddot{x}$ = resultant acceleration

and *m*, *c*, and *k* are the mass, damping constant, and spring constant, respectively. Equation (2) merely balances the inertia force ($m\ddot{x}$), the damping force ($c\dot{x}$), and the spring force (kx), against the externally applied force, $f(t)$.

The multiple degree of freedom case follows the same general procedure. Again, applying Newton's second law, one may write the equations of motion as:

$$m_1\ddot{x}_1 + (c_1 + c_2)\dot{x}_1 + (k_1 + k_2)x_1 - c_2\dot{x}_2 - k_2x_2 = 0 \tag{3}$$

and

$$m_2\ddot{x}_2 + (c_2 + c_3)\dot{x}_2 + (k_2 + k_3)x_2 - c_2\dot{x}_1 - k_2x_1 = F(t) \tag{4}$$

It is often more convenient to write equations (3) and (4) in matrix form:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} (c_1 + c_2) & (-c_2) \\ (-c_2) & (c_2 + c_3) \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} (k_1 + k_2) & (-k_2) \\ (-k_2) & (k_2 + k_3) \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ F(t) \end{Bmatrix} \tag{5}$$

or equivalently, for the general *n*-degree of freedom system,

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F\} \tag{6}$$

Where, $[M]$ = mass matrix, *(n x n),*
$[C]$ = damping matrix, *(n x n),*
$[K]$ = stiffness matrix, *(n x n),*

and the previously defined force, displacement, velocity, and acceleration terms are now *n*-dimensional vectors.

The mass, stiffness, and damping matrices contain all of the necessary mass, stiffness, and damping coefficients such that the equations of motion yield the correct time response when arbitrary input forces are applied.

The time-domain behavior of a complex dynamic system represented by equation (6) is very useful information. However, in a great many cases, frequency domain information turns out to be even more valuable. For example, natural frequency is an important characteristic of a mechanical system, and this can be more clearly identified by a frequency domain representation of the data. The choice of domain is clearly a function of what information is desired.

One of the most important concepts used in digital signal processing is the ability to transform data between the time and frequency domains via the Fast Fourier Transform (FFT) and the Inverse FFT. The relationships between the time, frequency, and Laplace domains are well defined and greatly facilitate the process of implementing modal analysis on a digital Fourier analyzer. Remember that the Fourier and Laplace transforms are the mathematical tools that allow data to be transformed from one independent variable to another (time, frequency or the Laplace s-variable). The discrete Fourier transform is a mathematical tool which is easily implemented in a digital processor for transforming time-domain data to its equivalent frequency domain form, and vice versa. It is important to note that no information about a signal is either gained or lost as it is transformed from one domain to another.

The transfer (or characteristic) function is a good example of the versatility of presenting the same information in three different domains. In the time domain, it is the unit impulse response, in the frequency domain the frequency response function and in the Laplace or *s*-domain, it is the transfer function. Most importantly, all are transforms of each other.

Because we are concerned with the identification of modal parameters from transfer function data, it is convenient to return to the single degree of freedom system and write equation (2) in its equivalent transfer function form.

The Laplace Transform. Recall that a function of time may be transformed into a function of the complex variable *s* by:

$$F(s) = \int_0^{\infty} f(t)e^{-st}dt \tag{7}$$

The Laplace transform of the equation of motion of a single degree of freedom system, as given in equation (2), is

$$m\left[s^2 X(s) - sx(0) - \dot{x}(0)\right] + c\left[sX(s) - x(0)\right] + kX(s) = F(s) \tag{8}$$

where,

$x(0)$ is the initial displacement of the mass *m* and

$\dot{x}(0)$ is the initial velocity.

This transformed equation can be rewritten by combining the initial conditions with the forcing function, to form a new $F(s)$:

$$\left[ms^2 + cs + k\right] X(s) = F(s) \tag{9}$$

It should now be clear that we have transformed the original ordinary differential equation into an algebraic equation where s is a complex variable known as the Laplace operator. It is also said that the problem is transformed from the time (real) domain into the s (complex) domain, referring to the fact that time is always a real variable, whereas the equivalent information in the *s*-domain is described by complex functions. One reason for the transformation is that the mathematics are much easier in the *s*-domain. In addition, it is generally easier to visualize the parameters and behavior of damped linear sustems in the *s*-domain.

Solving for X(s) from equation (9), we find

$$X(s) = \frac{F(s)}{ms^2 + cs + k} \tag{10}$$

The denominator polynomial is called the characteristic equation, since the roots of this equation determine the character of the time response. The roots of this characteristic equation are also called the *poles* or *singularities* of the system. The roots of the numerator polynomial are called the *zeros* of the system. Poles and zeros are critical frequencies. At the poles the function *x(s)* becomes infinite; while at the zeros, the function becomes zero. A transfer function of a dynamic system is defined as the ratio of the output of the system to the input in the *s*-domain. It is, by definition, a function of the complex variable *s*. If a system has *m* inputs and *n* resultant outputs, then the system has *m x n* transfer functions. The transfer function which relates the displacement to the force is referred to as the compliance transfer function and is expressed mathematically as,

$$H(s) = \frac{X(s)}{F(s)} \tag{11}$$

From equations (10) and (11), the compliance transfer function is,

$$H(s) = \frac{1}{ms^2 + cs + k} \tag{12}$$

Note that since s is complex, the transfer function has a real and an imaginary part. The Fourier transform is obtained by merely substituting $j\omega$ for *s*. This special case of the transfer Unction is called the *frequency response function*. In other words, the Fourier transform is merely the Laplace transform evaluated along the $j\omega$, or frequency axis, of the complex Laplace plane.

The analytical form of the frequency response function is therefore found by letting *s* = $j\omega$:

$$H(j\omega) = \frac{1}{-m\omega^2 + jc\omega + k} \tag{13}$$

By making the following substitutions in equation (13),

$$\omega_n^2 = \frac{k}{m}, \quad \zeta = \frac{c}{C_c} = \frac{c}{2\sqrt{km}}$$

C_c = critical damping coefficient

we can write the classical form of the frequency response function so,

$$\frac{X(j\omega)}{F(j\omega)} = H(j\omega) = \frac{1}{k\left[1 + 2\zeta j\left(\frac{\omega}{\omega_n}\right) - \frac{\omega^2}{\omega_n^2}\right]} \tag{14}$$

However, for our purposes, we will continue to work in the *s*-domain. The above generalized transfer function, equation (12), was developed in terms of compliance. From an experimental viewpoint, other very useful forms of the transfer function are often used and, in general, contain the same information. Table I summarizes these different forms.

Table I – Different forms of the transfer function for mechanical systems.

$\frac{\text{Displacement}}{\text{Force}}$ =	Dynamic Compliance	$\frac{\text{Force}}{\text{Displacement}}$ =	Dynamic Stiffness

$\frac{\text{Velocity}}{\text{Force}}$ =	Mobility	$\frac{\text{Force}}{\text{Velocity}}$ =	Mechanical Impedance
$\frac{\text{Acceleration}}{\text{Force}}$ =	Inertance	$\frac{\text{Force}}{\text{Acceleration}}$ =	Dynamic Mass

The *s*-Plane. Since s is a complex variable, we can represent all complex values of s by points in a plane. Such a plane is referred to as the *s*-plane. Any complex value of s may be located by plotting its real component on one axis and its imaginary component on the other. Now, the magnitude of any function, such as the compliance transfer function, $H(s)$, can be plotted as a surface above the plane of Figure 4. This requires a three-dimensional figure which can be difficult to sketch, but greatly facilitates the understanding of the transfer function. By definition, $s=\sigma+j\omega$ where σ is the *damping coefficient* and ω is the *angular frequency*.

The inertance transfer function of a simple two degree of freedom system is plotted as a function of the *s* variable in Figure 5. The transfer function evaluated along the frequency axis $(s=j\omega)$ is the Fourier transform or the system frequency response function. It is shown by the heavy line. If we were to measure the frequency response function for this system via experimental measurements using the Fourier transform, we would obtain a complex-valued function of frequency. It must be represented by its real (coincident) part and its imaginary (quadrature) part; or equivalently, by its magnitude and phase. These forms are shown in Figure 6.

In general, complex mechanical systems contain many modes of vibration or "degrees of freedom." Modern modal analysis techniques can be used to extract the modal parameters of each mode without requiring each mode to be isolated or excited by itself.

Modes of Vibration The equations of motion of an *n* degree of freedom system can be written as

$$B(s)\ X(s)=F(s) \qquad (15)$$

Where, $F(s)$ = Laplace transform of the applied force vector
$X(s)$ = Laplace transform of the resulting output vector
$B(s)=Ms^2+Cs+K$
s = Laplace operator

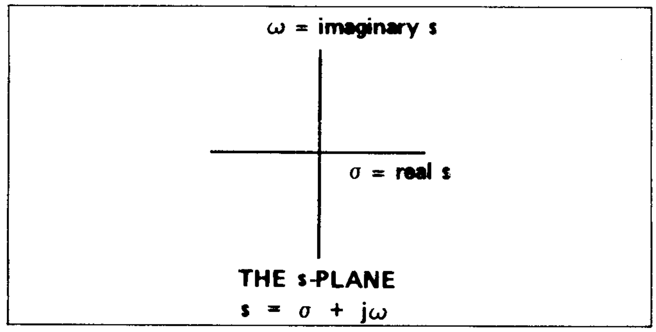


Figure 4—The s plane, $s=\sigma+j\omega$

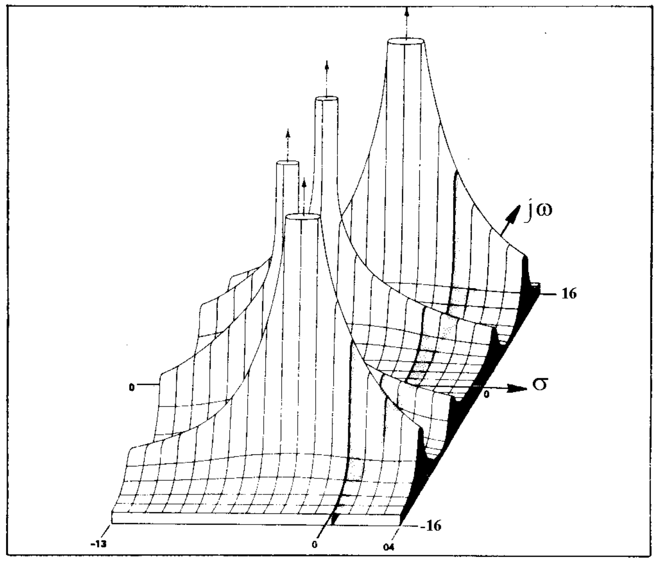


Figure 5—The magnitude of the Laplace transform of a two degree of freedom system with poles at $s=-3\pm j11$ *and* $s=-6\pm j6$.

$B(s)$ *is referred to as the system matrix. The transfer matrix,* $H(s)$ is defined as the inverse of the system matrix, hence it satisfies the equation.

$$X(s)=H(s)\ F(s) \qquad (16)$$

Each element of the transfer matrix is a transfer function.

From the general form of the transfer function described in equation (16), $H(s)$ can always be written in partial fraction form as:

$$H(s)=\sum_{k=1}^{2n}\frac{a_k}{s-p_k} \qquad (17)$$

where, n = number of degrees of freedom
p_k = k_{th} root of the equation obtained by setting the determinant of the matrix $B(s)$ equal to zero
a_k = residue matrix for the kit root.

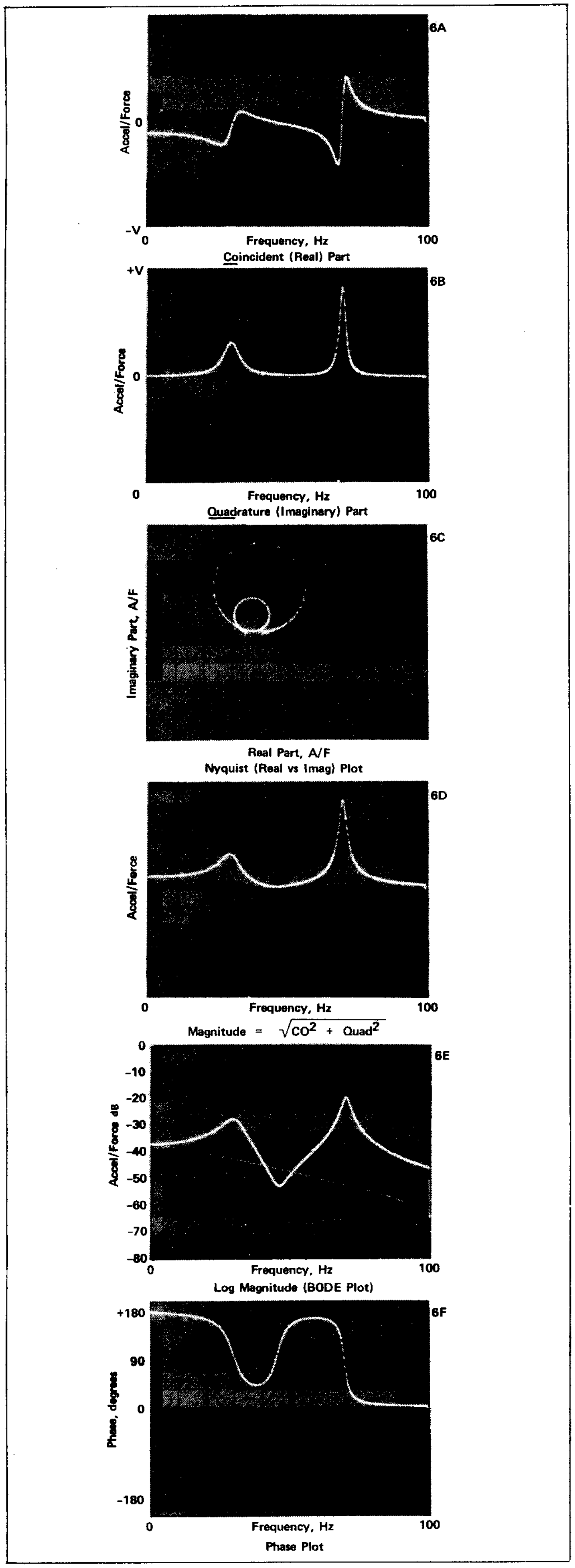


Figure 6—Frequency response functions of a two degree of freedom system.

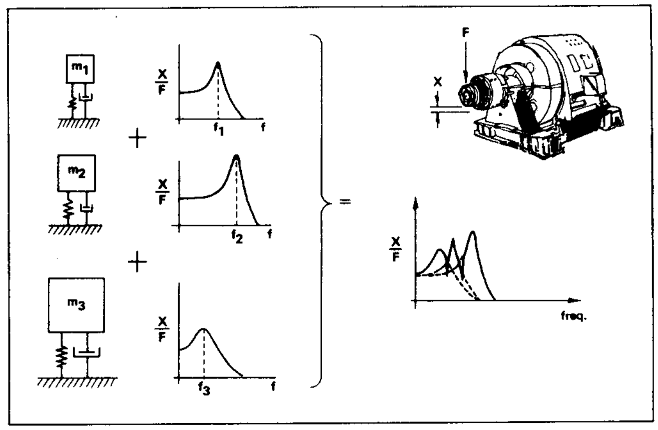


Figure 7—A multi-degree of freedom system can be ideally represented as a series of coupled single degree of freedom systems.

As mentioned earlier, the roots p_k are referred to as poles of the transfer function. These poles are complex numbers and always occur in complex conjugate pairs, except when the system is critically or supercritically damped. In the latter cases, the poles are real-valued and lie along the real (or damping) axis in the *s*-plane.

Each complex conjugate pair of poles corresponds to a *mode of vibration* of the structure. They are complex numbers written as

$$p_k = -\sigma_k + j\omega_k \quad p_k^* = -\sigma_k - j\omega_k \tag{18}$$

Where * denotes the conjugate, σ_k is the *modal damping coefficient*, and ω_k is the *natural frequency*. These parameters are shown on the *s*-plane in Figure 8. An alternate set of coordinates for defining the pole locations are the *resonant frequency*, given by

$$\Omega_k = \sqrt{\sigma_k^2 + \omega_k^2} \tag{19}$$

and the *damping factor*, or *percent of critical damping*, given by:

$$\zeta_k = \frac{\sigma_k}{\Omega_k} \tag{20}$$

The transfer matrix completely defines the dynamics of the system. In addition to the poles of the system (which define the natural frequency and damping), the residues from any row or column of $H(s)$ define the system

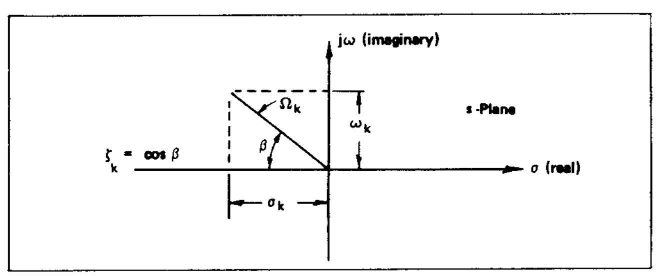


Figure 8—An s-plane representation for a single degree of freedom system.

mode shapes for the various natural frequencies. In general, a pole location, P_k, will be the same for all transfer functions in the system because a mode of vibration is a global property of an elastic structure. The values of the residues, however, depend on the particular transfer function being measured. The values of the residues determine the amplitude of the resonance in each transfer function and, hence, the mode shape for the particular resonance. From complex variable theory, we know that if we can measure the frequency response function (via the Fourier transform) then we know the exact form of the system (its transfer function) in the s-plane, and hence we can find the four important properties of any mode. Namely, its natural frequency, damping, and magnitude and phase of its residue or amplitude.

While this is a somewhat trivial task for a single degree of freedom system, it becomes increasingly difficult for complex systems with many closely coupled modes. However, considerable effort has been spent in recent years to develop sophisticated algorithms for curve-fitting to experimentally measured frequency response functions.[1,2] This allows the modal properties of each measured mode to be extracted in the presence of other modes.

From a testing standpoint, these new techniques offer important advantages. Writing equation (16) in matrix form gives:

$$\begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} = \begin{bmatrix} h_{11}(s) & h_{12}(s) \\ h_{21}(s) & h_{22}(s) \end{bmatrix} \begin{bmatrix} F_1(s) \\ F_2(s) \end{bmatrix} \qquad (21)$$

If only one mode is associated with each pole, then it can be shown that the modal parameters can be identified from *any* row or column of the transfer function matrix [*H*], except those corresponding to components known as node points. In other words, it is impossible to excite a mode by forcing it at one of its node points (a point where no response is present). Therefore, only one row or column need be measured.

To measure one column on the transfer matrix, an exciter would be attached to the structure (point #1 to measure column #1; point #2 to measure column #2) and responses would be measured at points #1 and #2. Then the transfer function would be formed by computing,

$$H(j\omega) = \frac{X(j\omega)}{F(j\omega)} \qquad (22)$$

To measure a row of the transfer matrix, the structure would be excited at point #1 and the response measured at point #1. Next, the structure would be excited at point #2 and the response again measured at point #1. This latter case corresponds to having a stationary response transducer at point #1, and using an instrumented hammer for applying impulsive forcing functions. Both of these methods are referred to as single point excitation techniques.

Complex Mode Shapes. Before leaving the structural dynamic model, it is important to introduce the idea of a complex mode shape. Without placing restrictions on damping beyond the fact that the damping matrix be symmetric and real valued, modal vectors can in general be complex valued. When the mode vectors are real valued, they are the equivalent of the *mode shape*. In the case of complex modal vectors, the interpretation is slightly different.

Recall that the transfer matrix for a single mode can be written as;

$$H_k(s) = \frac{a_k}{s - p_k} + \frac{a_k^*}{s - p_k^*} \qquad (23)$$

where

a_k = (*n x n*) complex residue matrix.

p_k = pole location of mode *k*.

A single component of $H(s)$ is thus written as

$$H(s) = \frac{r_k}{2j(s - p_k)} - \frac{r_k^*}{2j(s - p_k^*)} \qquad (24)$$

where

$\frac{r_k}{2j}$ = complex residue of mode *k*.

Now, the inverse Laplace transform of the transfer function of equation (24) is the *impulse response* of mode *k*; that is, if only mode *k* was excited by a unit impulse, its time domain response would be

$$x_k(t) = |r_k| e^{-\sigma_k t} \sin(\omega_k t - \alpha_k) \qquad (25)$$

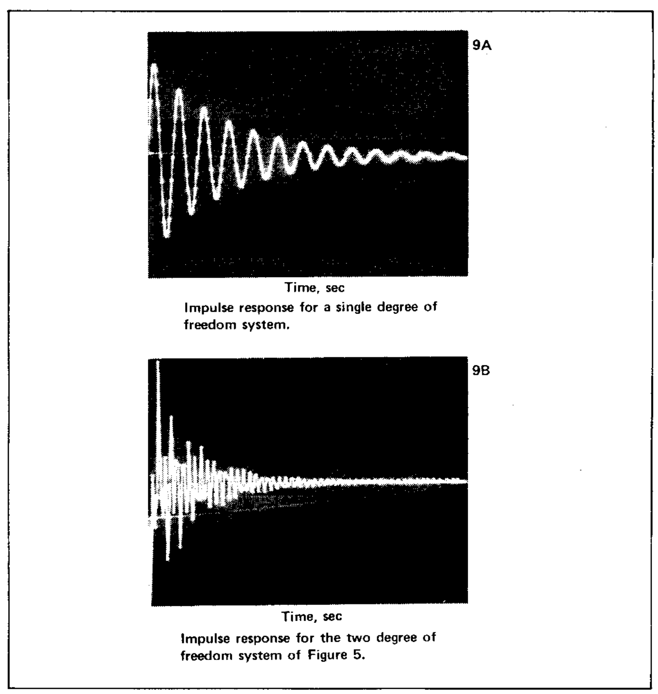


Figure 9—Impulse response for a single degree of freedom system and for the two degree of freedom system shown in Figure 5.

where

$|r_k|$ = magnitude of the residue

α_k = phase angle of the residue

A phase shift in the impulse response is introduced by the phase angle α_k of the complex residue. For $\alpha_k = 0$, the mode is said to be "normal" or real valued. It is this phase delay in the impulse response that is represented by the complex mode shape. Experimentally, a real or normal mode is characterized by the fact that all points on the structure reach their maximum or minimum deflection at the same time. In other words, all points are either in phase or 180° out of phase. With a complex mode, phases other than 0° and 180° are possible. Thus, nodal lines will be stationary for normal modes and nonstationary or "traveling" for complex modes. The impulse response for a single degree of freedom system and for the two degree of freedom system represented in Figure 5 are shown in Figure 9.

The digital Fourier Analyzer has proven to be an ideal tool for measuring structural frequency response functions quickly and accurately. Since it provides a broadband frequency spectrum very quickly (e.g., = 100 ms for 512 spectral lines when implemented in microcode), it can be used for obtaining broadband response spectrums from a structure which is excited by a broadband input signal. Furthermore, if the input and response time signals are measured simultaneously, Fourier transformed, and the transform of the response is divided by the transform of the input, a transfer function between the input and response points on the structure is measured. Because the Fourier Analyzer contains a digital processor, it possesses a high degree of flexibility in being able to post-process measured data in many ways.

It has been shown[1,2] that the modes of vibration of an elastic structure can be identified from transfer function measurements by the application of digital parameter identification techniques. Hewlett-Packard has implemented these techniques on the HP 5451B Fourier Analyzer. The system uses a single point excitation technique. This approach, when coupled with a broadband excitation allows all modes in the bandwidth of the input energy to be excited simultaneously. The modal frequencies, damping coefficients, and residues (eigenvectors) are then extracted from the measured broadband transfer functions via an analytical curve-fitting algorithm. This method thus permits an accurate definition of modal parameters without exciting each mode individually. Part II of this article will address the problem of making transfer function measurements.

The data shown in Figure 10 was obtained by using the Hewlett-Packard HP 5451B Fourier Analyzer to measure the required set of frequency response functions from a simple rectangular plate and identify the predominant modes of vibration. Figure 10A shows a typical frequency response function obtained from using an impulse testing technique on a flat aluminum

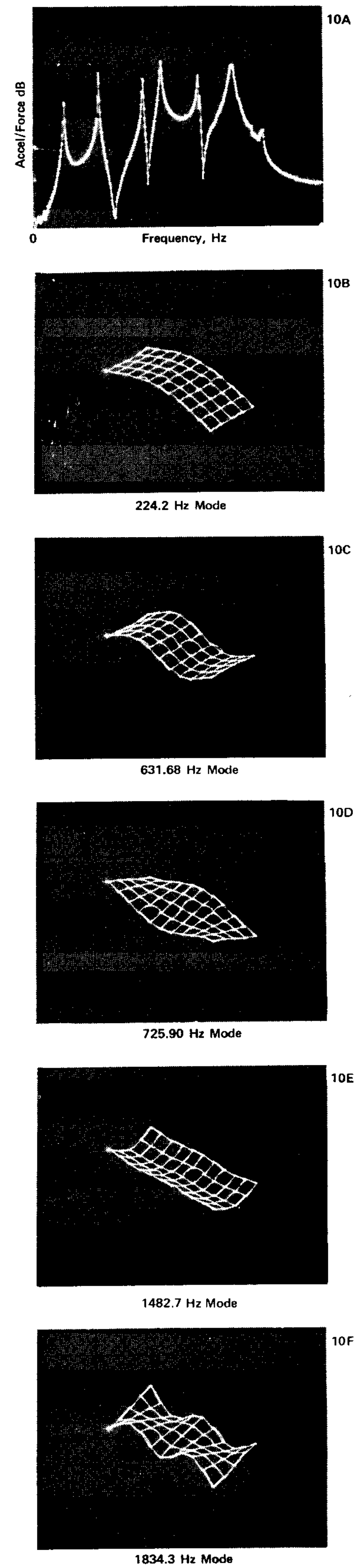


Figure 10—Typical frequency response function of the vibration of a simple rectangular plate (A) and animated isometric displays of the predominant vibration modes (B-F).

plate. Input force was measured with a load cell and the output response was measured with an accelerometer. After 55 such functions were measured and stored, the modal parameters were identified via a curve-fitting algorithm. In addition, the Fourier Analyzer provided an animated isometric display of each mode, the results of which are shown in Figures 10B – 10F.

The Transfer and Coherence Functions

The measurement of structural transfer functions using digital Fourier analyzers has many important advantages for the testing laboratory. However, it is imperative that one have a firm understanding of the measurement process in order to make effective measurements. For instance, digital techniques require that all measurements be discrete and of finite duration. Thus, in order to implement the Fourier transform digitally, it must be changed to a finite form known as the Discrete Fourier Transform (DFT). This means that all continuous time waveforms which must be transformed must be sampled (measured) at discrete intervals of time, uniformly separated by an interval Δt. It also means that only a finite number of samples *N* can be taken and stored. The record length *T* is then

$$T = N\Delta t \tag{26}$$

The effect of implementing the DFT in a digital memory is that it no longer contains magnitude and phase information at all frequencies as would be the ease for the continuous Fourier transform. Rather, it describes the spectrum of the waveform at discrete frequencies and with finite resolution up to some maximum frequency, $\mathrm{F_{max}}$, which according to Shannon's sampling theorem, obeys

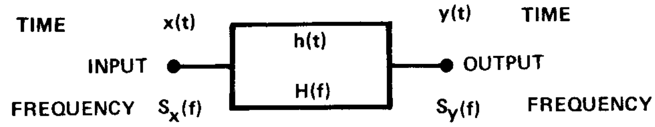
$$F_{\max} = \frac{1}{2\Delta t} \tag{27}$$

As a direct consequence of equation (27), we can write the physical law which defines the maximum frequency resolution obtainable for a sampled record of length, T.

$$\Delta f = \frac{1}{T} \tag{28}$$

When dealing with real valued-time functions, there will be *N* points in the record. However, to completely describe a given frequency, two values are required; the magnitude and phase or, equivalently, the real part and the imaginary part. Consequently, *N* points in the time domain can yield $N/2$ complex quantities in the frequency domain. With these important relationships in mind, we can return to the problem of measuring transfer functions.

The general case for a system transfer function measurement is shown below



where:

$x(t)$ = Time-domain input to the system
$y(t)$ = Time-domain response of the system
$S_x(f)$ = Linear Fourier spectrum of $x(t)$
$S_y(f)$ = Linear Fourier spectrum of $y(t)$
$H(f)$ = System transfer function (frequency domain)
$h(t)$ = System impulse response

The linear Fourier spectrum is a complex valued function that results from the Fourier transform of a time waveform. Thus, S_x and S_y have a real (in phase or coincident) and imaginary (quadrature) parts.

In general, the result of a linear system on any time domain input signal, $x(t)$, may be determined from the convolution of the system impulse response, $h(t)$, with the input signal, $x(t)$, to give the output, $y(t)$.

$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau \tag{29}$$

This operation may be difficult to visualize. However, a very simple relationship can be obtained by applying the Fourier transform to the convolution integral. The output spectrum, S_y, is the product of the input spectrum, S_x, and the system transfer function, $H(f)$.

$$S_y(f) = S_x(f) \cdot H(f) \tag{30}$$

In other words, the transfer function of the system is defined as:

$$H = \frac{\mathrm{OUTPUT}}{\mathrm{INPUT}} = \frac{S_y}{S_x} \tag{31}$$

The simplest implementation of a measurement scheme based on this technique is the use of a sine wave for $x(f)$. However, in many cases, this signal has disadvantages compared to other more general types of signals. The most general method is to measure the input and output time waveforms in whatever form they may be, and to calculate *H* using S_x, S_y and the Fourier transform.

For the general measurement case, the input $x(t)$ is not sinusoidal and will often be chosen to be random noise, especially since it has several advantages when used as a stimulus for measuring structural transfer functions. However, it is not generally useful to measure the linear spectrum of this type of signal because it cannot be smoothed by averaging; therefore we typically resort to the power spectrum.

The power spectrum of the system input is defined and computed as:

$$\begin{aligned} G_{xx} &= \text{Power spectrum of the input } x(t) \\ &= S_x S_x^* \end{aligned} \tag{32}$$

where

S_x^* = Complex conjugate of S_x.

and

$$\begin{aligned} G_{yy} &= \text{Power spectrum of the output } y(t) \\ &= S_y S_y^* \end{aligned} \tag{33}$$

where

S_y^* = Complex conjugate of S_y.

The cross power spectrum between the input and the output is denoted by G_{yx} and defined as,

$$G_{yx} = S_y S_x^* \tag{34}$$

Returning to equation (31), we can multiply the numerator and denominator by S_x^*. This shows that the transfer function can be expressed as the ratio of the cross power spectrum to the input auto power spectrum.

$$H = \frac{S_y}{S_x} \bullet \frac{S_x^*}{S_x^*} = \frac{G_{yx}}{G_{xx}} \tag{35}$$

There are three important reasons for defining the system transfer function in this way. First, this technique measures magnitude and phase since the cross power spectrum contains phase information. Second, this formulation is not limited to sinusoids, but may in fact be used for any arbitrary waveform that is Fourier transformable (as most physically realizable time functions are). Finally, averaging can be applied to the measurement. This alone is an important consideration because of the large variance in the transfer function estimate when only one measurement is used. So, in general,

$$H(f) = \frac{\overline{G_{yx}}}{\overline{G_{xx}}} \tag{36}$$

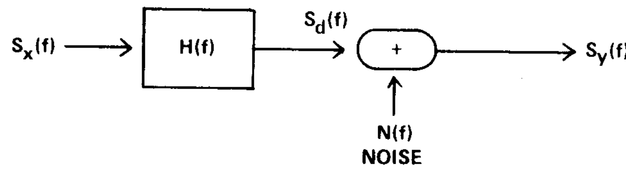
where $\overline{G_{yx}}$ denotes the ensemble average of the cross power spectrum and $\overline{G_{xx}}$ represents the ensemble average of the input auto power spectrum.

As an added note, the impulse response $h(t)$ of a linear system is merely the inverse transform of the system transfer function,

$$h(t) = F^{-1} \left\{ \frac{\overline{G_{yx}}}{\overline{G_{xx}}} \right\} \tag{37}$$

Reducing Measurement Noise

The importance of averaging becomes much more evident if the transfer function model shown above is expanded to depict the "real-world" measurement situation. One of the major characteristics of any modal testing system is that extraneous noise from a variety of sources is always measured along with the desired excitation and response signals. This case for transfer function measurements is shown below.



where:

$S_x(f)$ = Lineal Fourier spectrum of the measured input signal

$S_d(f)$ = Linear Fourier spectrum of the desired response measurement

$S_y(f)$ = Linear Fourier spectrum of the measured response

$N(f)$ = Linear Fourier spectrum of the noise

$H(f)$ = System transfer function

Since we are interested in identifying modal parameters from measured transfer functions, the variance on the parameter estimates is reduced in proportion to the amount of noise reduction in the measurements. The digital Fourier analyzer has two inherent advantages over other types of analyzers in reduction of measurement noise; namely, ensemble averaging, and a second technique commonly referred to as post data smoothing which may be applied after the measurements are made.

Without repeating the mathematics for the general model of a transfer function measurement in the presence of noise, it is easy to show that the transfer function is more accurately written as:

$$H = \frac{\overline{G_{yx}}}{\overline{G_{xx}}} - \frac{\overline{G_{nx}}}{\overline{G_{xx}}} \tag{38}$$

where the frequency dependence notation has been dropped and,

$\overline{G_{yx}}$ = Ensemble average of the cross power spectrum between input and output

$\overline{G_{xx}}$ = Ensemble average of the input power spectrum

$\overline{G_{nx}}$ = Ensemble average of the cross power spectrum between the noise and the input

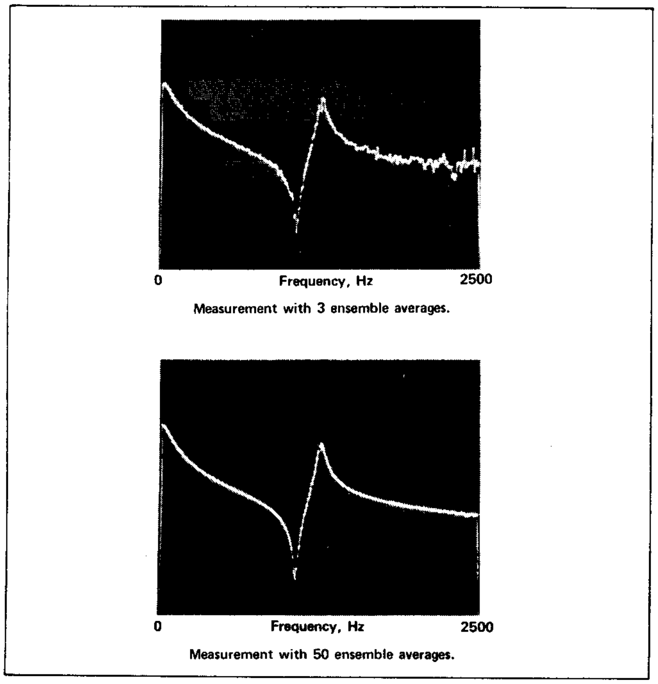


Figure 11—Effect of averaging on transfer function measurements.

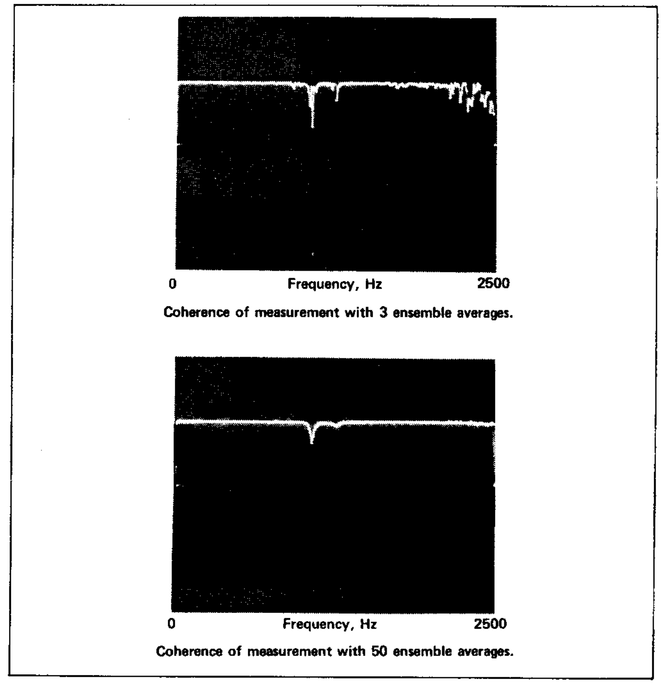


Figure 12—Effect of ensemble averaging on the coherence for the transfer function shown in Figure 11.

This form assumes that the noise has a zero mean value and is incoherent with the measured input signal. Now, as the number of ensemble averages becomes larger, the noise term $\overline{G_{nx}}$ becomes smaller and the ratio $\overline{G_{yx}} / \overline{G_{xx}}$ more accurately estimates the true transfer function. Figure 11 shows the effect of averaging on a typical transfer function measurement.

The Coherence Function

To determine the quality of the transfer function, it is not sufficient to know only the relationship between input and output. The question is whether the system output is totally caused by the system input. Noise and/or non-linear effects can cause large outputs at various frequencies, thus introducing errors in estimating the transfer function. The influence of noise and/or non-linearities, and thus the degree of noise contamination in the transfer function is measured by calculating the coherence function, denoted by γ^2, where

$$\gamma^2 = \frac{\text{Response power caused by applied input}}{\text{Measured response power}} \tag{39}$$

The coherence function is easily calculated on a digital Fourier analyzer when transfer functions are being measured. It is calculated as:

$$\gamma^2 = \frac{\left|\overline{G_{yx}}\right|^2}{\overline{G_{xx}}\,\overline{G_{yy}}} \quad \text{where} \quad 0 \le \gamma^2 \le 1 \tag{40}$$

If the coherence is equal to 1 at any specific frequency, the system is said to have perfect causality at that frequency. In other words, the measured response power is caused totally by the measured input power (or by sources which are coherent with the measured input power). A coherence value less than 1 at a given frequency indicates that the measured response power is greater than that due to the measured input because some extraneous noise is also contributing to the output power.

When the coherence is zero, the output is caused totally by sources other than the measured input. In general terms, the coherence is a measure of the degree of noise contamination in a measurement. Thus, with more averaging, the estimate of coherence contains less variance, therefore giving a better estimate of the noise energy in a measured signal. This is illustrated in Figure 12.

Since the coherence function indicates the degree of causality in a transfer function it has two very important uses:

1) It can be used qualitatively to determine how much averaging is required to reduce measurement noise.
2) It can serve as a monitor on the quality of the transfer function measurements.

The transfer functions associated with most mechanical systems are so complex in nature that it is virtually impossible to judge their validity solely by inspection. In one case familiar to the author, a spacecraft was being excited with random noise in order to obtain structural transfer functions for modal parameter identification. The transfer and coherence functions were monitored for each measurement. Then, between two measurements the coherence function became noticeably different from unity. After rechecking all instrumentation, it was discovered that a random vibration test being conducted in a separate part of the same building was providing incoherent excitation via structural (building) coupling, even through a seismic isolation mass. This extraneous source was increasing the variance on the measurement but would probably not have been discovered without use of the coherence function.

Summary

In Part I, we have introduced the structural dynamic model for elastic structures and the concept of a mode of vibration in the Laplace domain. This means of representing modes of vibration is very useful because we are interested in identifying the modal parameters from measured frequency response functions. Lastly, the procedure for calculating transfer and coherence functions in a digital Fourier analyzer were discussed.

In Part II, we will discuss various techniques for accurately measuring structural transfer functions. Because modal parameter identification algorithms work on actual measured data, we are interested in making the best measurements possible, thus increasing the accuracy of our parameter estimates. Techniques for exciting structures with various forms of excitation will be discussed. Also, we will discuss methods for arbitrarily increasing the available frequency resolution via band selectable Fourier analysis—the so-called zoom transform.

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Effective Measurements for Structural Dynamics Testing

Part II

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Digital Fourier analyzers have opened a new era in structural dynamics testing. The ability of these systems to measure a set of structural transfer functions quickly and accurately and then operate on them to extract modal parameters is having a significant impact on the product design and development cycle. In order to use these powerful new tools effectively, it is necessary to have a basic understanding of the concepts which are employed. In Part I of this article, the structural dynamics model was introduced and used for presenting the basic mathematics relative to modal analysis and the representation of modal parameters in the Laplace domain. Part I concluded with a section describing the basic theoretical concepts relative to measuring transfer and coherence functions with a digital Fourier analyzer. Part II presents an introductory discussion of several techniques for measuring structural transfer functions with a Fourier analyzer. Broadband testing techniques are stressed and digital techniques for identifying closely coupled modes via increased frequency resolution are introduced.

Certainly one of the most important areas of structural dynamics testing is the use of modern experimental techniques for modal analysis. The development of analytical and experimental methods for identifying modal parameters with digital Fourier analyzers has had a dramatic impact on product design in a number of industries. The application of these new concepts has been instrumental in helping engineers design mechanical structures which carry more payload, vibrate less, run quieter, fail less frequently, and generally behave according to design when operated in a dynamic environment.

Making effective measurements in structural dynamics testing can be a challenging task for the engineer who is new to the area of digital signal analysis. These powerful new signal analysis systems represent a significant departure from traditional analog instrumentation in terms of theory and usage. By their very nature, digital techniques require that all measurements be discrete and of finite duration, as opposed to continuous duration in the analog domain. However, the fact that digital Fourier analyzers utilize a digital processor enables them to offer capabilities to the testing laboratory that were unheard of only a few years ago.

Modal analysis, an important part of the overall structural dynamics problem, is one area that has benefited tremendously from the advent of digital Fourier analysis. The intent of this article is to present some of the important topics relative to understanding and making effective measurements for use in modal analysis. The engineer using these techniques needs to have a basic understanding of the theory on which the identification of modal parameters is based, in order to make a measurement which contains the necessary information for parameter extraction.

Part I of this article introduced the structural dynamics model and how it is represented in the Laplace or *s*-domain. The Laplace formulation was used, because it provides a convenient model to present the definition of modal parameters and the mathematics for describing a mode of vibration.

In this part, we will diverge from the mathematics and present some practical means for measuring structural transfer functions for the purpose of modal parameter identification. Unfortunately, the scope of this article does not permit a thorough explanation of many factors which are important to the measurement process, such as sampling, aliasing, and leakage.[1] Instead, we will concentrate more on different types of excitation and the importance of adequate frequency resolution.

Identification of Modal Parameters: a Short Review

In Part I we derived the time, frequency and Laplace or *s*-plane representation of a single-degree-of-freedom system, which has only one mode of vibration.

The time domain representation is a statement of Newton's second law

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = f(t) \tag{1}$$

where

$f(t)$ = applied force
$x(t)$ = resultant displacement
$\dot{x}(t)$ = resultant velocity
$\ddot{x}(t)$ = resultant acceleration
m = mass
c = damping constant
k = spring constant

This equation of motion gives the correct time domain response of a vibrating system consisting of a single mass, spring and damper, when an arbitrary input force is applied

The *transfer function* of the single-degree-of-freedom system is derived in terms of its *s*-plane representation by introducing the Laplace transform. The transfer function is defined as the ratio of the Laplace transform of the output of the system to the Laplace transform of

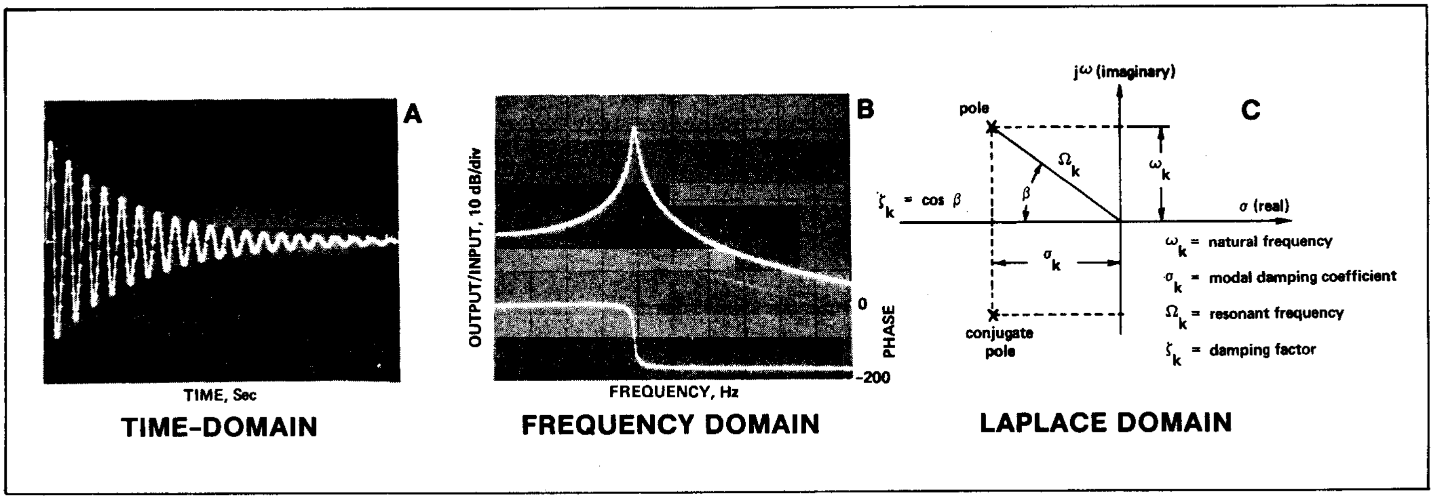

Figure I—A mechanical system can be described in: (A) the time domain, (B) the frequency domain or (C) the Laplace domain.

the input. The compliance transfer function was written as

$$H(s) = \frac{X(s)}{F(s)} = \frac{1}{ms^2 + cs + k} \qquad (2)$$

Finally, the Frequency domain form is found by applying the fact that the Fourier transform is merely the Laplace transform evaluated along the $j\omega$ or frequency axis ot the complex Laplace plane. This special case of the transfer function is called the *frequency response function* and is written as,

$$H(j\omega) = \frac{X(j\omega)}{F(j\omega)} = \frac{1}{k\left[1 + 2\zeta j\left(\frac{\omega}{\omega_n}\right) - \left(\frac{\omega^2}{\omega_n^2}\right)\right]} \qquad (3)$$

where:

$$\omega_n^2 = \frac{k}{m}$$

$$\zeta = \frac{c}{c_c} = \frac{c}{2\sqrt{km}}$$

C_c = critical damping

ω_n = natural frequency

Thus, as shown in Figure 1, the motion of a mechanical system can be completely described as a function of time, frequency, or the Laplace variable, *s*. Most importantly, all are valid ways of characterizing a system and the choice generally dictated by the type of information that is desired.

Because the behavior of mechanical structures is more easily characterized in the frequency domain, especially in terms of modes of vibration, we will devote our attention to their frequency domain description. A mode of vibration (the k^{th} mode) is completely described by the four Laplace parameters: ω_k, *the natural frequency;* σ_k, *the modal damping co-efficient;* and the *complex residue*, which is expressed as two terms, magnitude and phase. The residues define the *mode shapes* for the system. The Fourier transform is the tool that allows us to transform time domain signals to the Frequency domain and thus observe the Laplace domain along the frequency axis. It is possible to show that the transfer function over the entire *s*-plane is completely determined by its values along the $j\omega$ axis, so the frequency response function contains all of the necessary information to identify modal parameters.

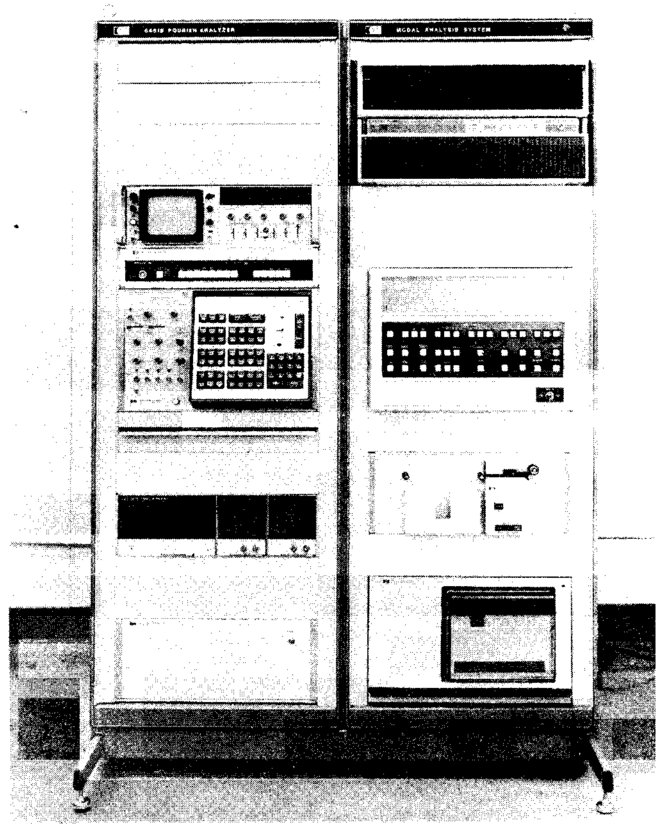

Figure 2—The Hewlett-Packard 5451B FourierAnalyzer is typical of modern digital signal analyzers which are being increasingly used for the acquisition and processing of modal analysis data.

Digital Fourier analyzers, such as the one shown in Figure 2, have proven to be ideal tools for measuring structural frequency response functions (transfer functions) quickly and accurately. Coupling this with the fact that modes of vibration can be identified from measured frequency respouse functions by digital parameter identification techniques gives the testing laboratory an accurate and cost-effective means for quickly characterizing a structure's dynamic behavior by identifying its modes of vibrations.[2]

The remainder of this article will attempt to introduce some of the techniques which are available for making effective frequency response measurements with digital Fourier analyzers.

Measuring Structural Frequency Response Functions

The general scheme for measuring frequency response functions with a Fourier analyzer consists of measuring simultaneously an input and response signal in the time domain, Fourier transforming the signals, and then forming the system transfer function by dividing the transformed response by the transformed input. This digital process enjoys many benefits over traditional analog techniques in terms of speed, accuracy and post-processing capability.[3] One of the most important features of Fourier analyzers is their ability to form accurate transfer functions with a variety of excitation methods. This is in contrast to traditional analog techniques which utilize sinusoidal excitation. Other types of excitation can provide faster measurements and a more accurate simulation of the type of excitation which the structure may actually experience in service. The only requirement on excitation functions with a digital Fourier analyzer is that they contain energy at the frequencies to be measured.

The following sections will discuss three popular methods for exciting a structure for the purpose of measuring transfer functions; they are, random, transient, and sinusoidal excitation. To begin with, we will restrict our discussion to baseband measurements; i.e., measurements made from dc (zero frequency) to some $F_{\max}$ (maximum requency). The procedures for using these broadband stimuli (except transient) are all very similar. They are typically used to drive a shaker which in turn excites the mechanical structure under test. The general process is illustrated in Figure 3.

Random Excitation Techniques

In this section, three types of broadband random excitation which can be used for making frequency response measurements are discussed. Each one possesses a distinct set of characteristics which should be understood in order to use them effectively. The three types are: (1) pure random, (2) pseudo random, and (3) periodic random.

Typically, pure random signals are generated by an external signal generator, whereas pseudo random and periodic random are generated by the analyzer's processor and output to the structure via a

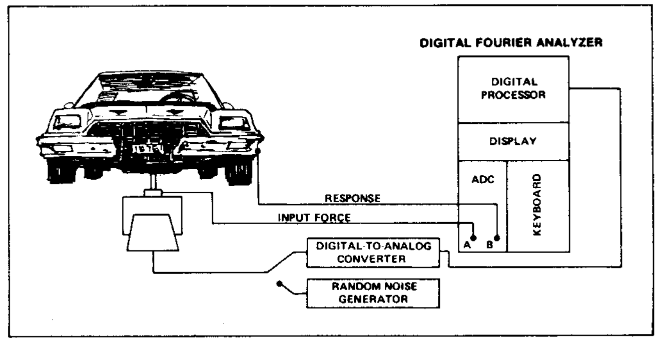

Figure 3—The general test setup for rnaking frequency response measurements with a digital Fourier analyzer and an electro-dynamic shaker.

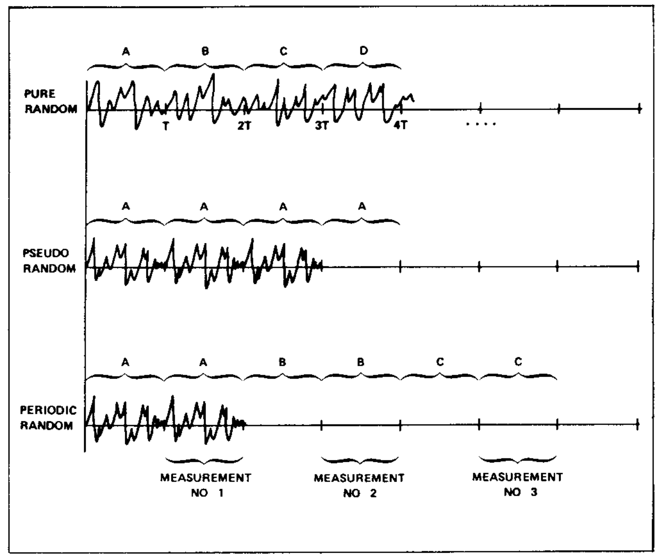

Figure 4—Comparison of pure random, pseudo random, and periodic random noise. Pure random is never periodic. Pseudo random is exactly periodic every T seconds. Periodic random is a combination of both; i.e., a pseudo random signal that is changed for every ensemble average.

digital-to-analog coverter, as shown in Figure 3. Figure 4 illustrates each type of random signal.

Pure Random

Pure random excitation typically has a Gaussian distribution and is characterized by the fact that it is in no way periodic, i.e., does not repeat. Typically, the output of an independent signal generator may he passed through a bandpass filter in order to concentrate energy in the band of interest. Generally, the signal spectrum will be flat except for the filter roll-off and, hence, only the overall level is easily controlled.

One disadvantage of this approach is that, although the shaker is being driven with a flat input spectrum, the structure is being excited by a force with a different spectrum due to the impedance mismatch between the structure and shaker head. This means that the force spectrum is not easily controlled and the structure may not be forced in the optimum manner. Since it is difficult to shape the spectrum because it is not generally controlled by the computer, some form of closed-loop force control system would ideally be used. Fortunately, in most cases, the problem is not important enough to warrant this effort.

A more serious drawback of pure random excitation is that the measured input and response signals are not periodic in the measurement time window of the analyzer. A key assumption of digital Fourier analysis is that the time waveforms be exactly periodic in the observation window. If this condition is not met, the corresponding frequency spectrum will contain so-called "leakage" due to the nature of the discrete Fourier transform; that is, energy from the non-periodic parts of the signal will "leak" into the periodic parts of the spectrum, thus giving a less accurate result.[1]

In digital signal analyzers, non-periodic time domain data is typically multiplied by a weighting function such as a Hanning window to help reduce the leakage caused by non-periodic data and a standard rectangular window.

When a non-periodic time waveform is multiplied by this window, the values of the signal in the measurement window more closely satisfy the requirements of a periodic signal. The result is that leakage in the spectrum of a signal which has been multiplied by a Hanning window is greatly reduced.

However, multiplication of two time waveforms, i.e., the non-periodic signal and the Hanning window, is equivalent to the convolution of their respective Fourier transforms (recall that multiplication in one domain is exactly equivalent to convolution in the other domain). Hence, although multiplication of a non-periodic signal by a Hanning window reduces leakage, the spectrum of the signal is still distorted due to the convolution with the Fourier transform of the Hanning window. Figure 5 illustrates these points for a simple sinewave.

With a pure random signal, each sampled record of data T seconds long is different from the proceeding and following records. (Figure 4). This gives rise to the single most important advantage of using a pure random signal for transfer function measurement. Successive records of frequency domain data can be ensemble averaged together to remove non-linear effects, noise, and distortion from the measurement. As more and more averages are taken, all of these components of a structure's motion will average toward an expected value of zero in the frequency domain data. Thus, a much better measure of the linear least squares estimate of the response of the structure can be obtained.[3]

This is especially important because digital parameter estimation schemes are all based on linear models and the premise that the structure behaves in a linear manner. Measurements that contain distortion will be more difficult to handle if the modal parameter identification techniques used are based upon a linear model of the structure's motion.

Pseudo-Random

In order to avoid the leakage effects of a non-periodic signal, a waveform known as pseudo random is commonly used. This type of excitation is easy to implement with a digital Fourier analyzer and its

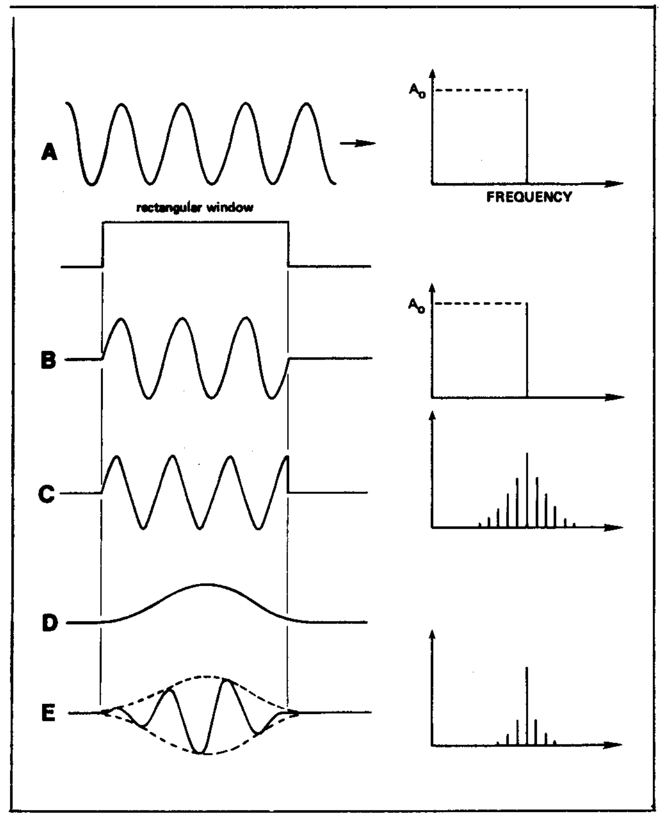

Figure 5—(A) A sinewave is continuous throughout time and is represented by a single line in the frequency domain; (B) when observed with a standard rectangular window, it is still a single spectral line, if it is exactly periodic in the window; (C) if it is not periodic in the measurement window, leakage occurs and energy "leaks" into adjacent frequency channels; (D) the Hanning window is one of many types of windows which are useful for reducing the effects of leakage; and (E) multiplying the time domain data by the Hanning window causes it to more closely meet the requirement of a periodic signal, thus reducing the leakage effect.

digital-to-analog (DAC) converter. The most commonly used pseudo random signal is referred to as "zero-variance random noise." It has uniform spectral density and random phase. The signal is generated in the computer and repeatedly output to the shaker through the DAC every T seconds (Figure 4). The length of the pseudo random record is thus exactly the same as the analyzer's measurement record length (T seconds), and is thus exactly periodic in the measurement window.

Because the signal generation process is controlled by the analyzer's computer, any signal which can be described digitally can be output through the DAC. The desired output signal is generated by specifying the amplitude spectrum in the frequency domain; the phase spectrum is normally random. The spectrum is then Fourier transformed to the time domain and output through the DAC. Therefore, it is relatively easy to alter the stimulus spectrum to account for the exciter system characteristics.

In general, besides providing leakage-free mesurements, a technique using pseudo random noise can often provide the fastest means for making a statistically accurate transfer function measurement when using a random stimulus. This proves to be the case when the measurement is relatively free of extraneous noise and the system behaves linearly, because the same signal is output repeatedly and large numbers of averages offer no significant advantages other than the reduction of extraneous noise.

The most serious disadvantage of this type of signal is that because it always repeats with every measurement record taken, non-linearities, distortion, and periodicities due to rattling or loose components on the structure cannot be removed from the measurement by ensemble averaging, since they are excited equally every time the pseudo random record is output.

Periodic Random

Periodic random waveforms combine the best features of pure random and pseudo random, but without the disadvantages; that is, it satisfies the conditions for a periodic signal, yet changes with time so that it excites the structure in a truly random manner.

The process begins by outputting a pseudo random signal from the DAC to the exciter. After the transient part of the excitation has died out and the structure is vibrating in its steady-state condition, a measurement is taken; i.e., input, output, and cross power spectrums are formed. Then instead of continuing to output the same signal again, a different uncorrelated pseudo random signal is synthesized and output (refer again to Figure 4). This new signal excites the structure in a new steady-state manner and another measurement is made.

When the power spectrums of these and many other records are averaged together, non-linearities and distortion components are removed from the transfer function estimate. Thus, the ability to use a periodic random signal eliminates leakage problems and ensemble averaging is now useful for removing distortion because the structure is subjected to a different excitation before each measurement.

The only drawback to this approach is that it is not as fast as pseudo random or pure random, since the transient part of the structure's response must be allowed to die out before a new ensemble average can be made. The time required to obtain a comparable number of averages may be anywhere from 2 to 3 times as long. Still, in many practical cases where a baseband measurement is appropriate, periodic random provides the best solution, in spite of the increased measurement time.

Sinusoidal Testing

Until the advent of the Fourier analyzer, the measure ment of transfer functions was accomplished almost exclusively through the use of swept-sine testing. With this method, a controlled sinusoidal force is input to the structure, and the ratio of output response to the input force versus frequency is plotted. Although sine testing was necessitated by analog instrumentation, it is certainly not limited to the analog domain. Sinusoidally measured transfer functions can be digitized and processed with the Fourier analyzer or can be measured directly, as we will explain here.

In general, swept sinusoidal excitation with analog instrumentation has several disadvantages which severely limit its effectiveness:

1) Using analog techniques, the low frequency range is often limited to several Hertz.
2) The data acquisition time can be long.
3) The dynamic range of the analog instrumentation limits the dynamic range of the transfer function measurements.
4) Accuracy is often difficult to maintain.
5) Non-linearities and distortion are not easily coped with.

However, swept-sine testing does offer some advantages over other testing forms:

1) Large amounts of energy can be input to the structure at each particular frequency.
2) The excitation force can be controlled accurately.

Being able to excite a structure with large amounts of energy provides at least two benefits. First, it results in relatively high signal-to-noise ratios which aid in determining transfer function accuracy and, secondly, it allows the study of structural non-linearities at any specific frequency, provided the sweep frequency can be manually controlled.

Sine testing can become very slow, depending upon the frequency range of interest and the sweep rate required to adequately define modal resonances. Averaging is accomplished in the time domain and is a function of the sweep rate.

A sinusoidal stimulus can be utilized in conjunction with a digital Fourier Analyzer in many different ways. However, the fastest and most popular method utilizes a type of signal referred to as a "chirp." A chirp is a logarithmically swept sinewave that is periodic in the analyzer's measurement window, T. The swept sine is generated in the computer and output through the DAC every T seconds. Figure 10G shows a chirp signal. The important advantage of this type of signal is that it is sinusoidal and has a good peak-to-rms ratio. This is an important consideration in obtaining the maximum accuracy and dynamic range from the signal conditioning electronics which comprise part of the test setup. Since the signal is periodic, leakage is not a problem. However, the chirp suffers the same disadvantage as a pseudo random stimulus; that is, its inability to average out non-linear effects and distortion.

Any number of alternate schemes for using sinusoidal excitation can be implemented on a Fourier analyzer. However, they will not be discussed here because they offer few, if any, advantages over the chirp and, in fact, generally serve to make the measurement process more tedious and lengthy.

Transient Testing

As mentioned earlier, the transfer function of a system can be determined using virtually any physically realizable input, the only criteria being that some input signal energy exists at all frequencies of interest. However, before the advent of mini-computer-based Fourier analyzers, it was not practical to determine the Fourier transform of experimentally generated input and response signals unless they were purely sinusoidal.

These digital analyzers, by virtue of the fast Fourier transform, have allowed transient testing techniques to become widely used. There are two basic types of transient tests: (1) Impact, and (2) Step Relaxation.

Impact Testing

A very fast method of performing transient tests is to use a hand-held hammer with a load cell mounted to it to impact the structure. The load cell measures the input force and an accelerometer mounted on the structure measures the response. The process of measuring a set of transfer functions by mounting a stationary response transducer (accelerometer) and moving the input force around is equivalent to attaching a mechanical exciter to the structure and moving the response transducer from point to point. In the former case, we are measuring a row of the transfer matrix whereas in the latter we are measuring a column.[2]

In general, impact testing enjoys several important advantages:

1) No elaborate fixturing is required to hold the structure under test.
2) No electro-mechanical exciters are required.
3) The method is extremely fast—often as much as 100 times as fast as an analog swept-sine test.

However, this method also has drawbacks. The most serious is that the power spectrum of the input force is not as easily controlled as it is when a mechanical shaker is used. This causes non-linearities to be excited and can result in some variablity between successive measurements. This is a direct consequence of the shape and amplitude of the input force signal.

The impact force can be altered by using a softer or harder hammer head. This, in turn, alters the corresponding power spectrum. In general, the wider the width of the force impulse, the lower the frequency range of excitation. Therefore, impulse testing is a matter of trade-offs. A hammer with a hard head can be used to excite higher frequency modes, whereas a softer head can be used to concentrate more energy at lower frequencies. These two cases are illustrated in Figures 6 and 7.

Since the total energy supplied by an impulse is distributed over a broad frequency range, the actual excitation energy density is often quite small. This presents a problem when testing large, heavily damped structures, because the transfer function estimate will suffer due to the poor signal-to-noise ratio of the measurement. Ensemble averaging, which can be used with this method, will greatly help the problem of poor signal-to-noise ratios.

Another major problem is that of frequency resolution. Adequate frequency resolution is an absolute necessity in making good structural transfer function measurements. The fundamental nature of a transient response signal places a practical limitation on the resolution obtainable. Inorder to obtain good frequency resolution for quantifying very lightly damped

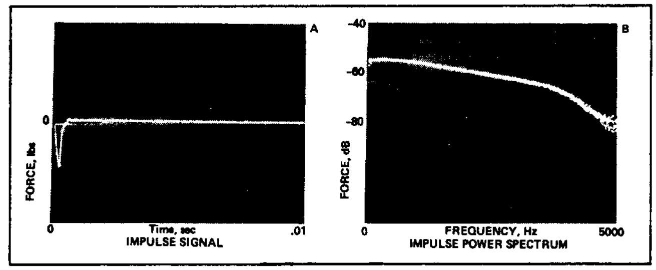

Figure 6—An instrumented hammer wih a hard head is used for exciting higher frequency modes but with reduced energy density.

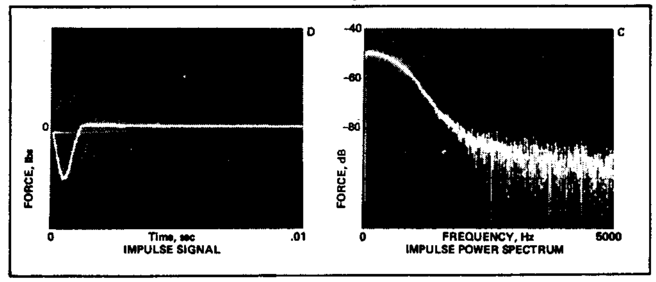

Figure 7—An instrumented hammer with a soft head can be used for concentrating more energy at lower frequencies, however, higher frequencies are not excited.

resonances, a large number of digital data points must be used to represent the signal. This is another way of saying that the Fourier transform size must be large since

$$\Delta f = \frac{\text{maximum frequency of interest}}{\frac{1}{2}\text{Fourier transform size}} = \frac{1}{T}$$

Thus, as the response signal decays to zero, its signal-to-noise ratio becomes smaller and smaller. If it has decayed to a small value before a data record is completely filled the Fourier transform will be operating mostly on noise therefore causing uncertainties in the transfer function measurement. Obviously, the problem becomes more acute as higher frequency resolutions are needed and as more heavily damped structures are tested. Figure 8 illustrates this case for a simple single-degree-of-freedom system. In essence frequency resolution and damping form the practical limitations for impulse testing with baseband (dc to $F_{\max}$) Fourier analysis.

Since a transient signal may or may not decay to zero within the measurement window, windowing can be a serious problem in many cases, especially when the damping is light and the structure tends to vibrate for a long time. In these instances, the standard rectangular window is unsatisfactory because of the severe leakage. Digital Fourier analyzers allow the user to employ a variety of different windows which will alleviate the problem. Typically, a Hanning window would be unsuitable because it destroys data at the first of the record--the most important part of a transient signal. The *exponential window* can be used to preserve the important information in the waveform while at the same time forcing the signal to become periodic. It must,

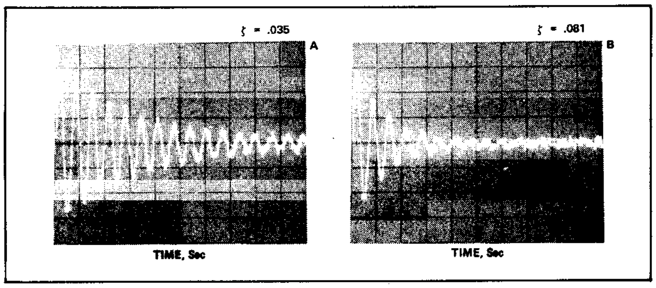

Figure 8—The impulse responses for two single-degree-of-freedom systems with different amounts of damping. Each measurement contains exactly the same amount of noise. The Fourier transform of the heavily damped system will have more uncertainty because of the poor signal-to-noise ratio in the last half of the data record.

however, be applied with care, especially when modes are closely spaced, for exponential smoothing can smear modes together so that they are on longer discernible as separate modes. Reference 4 explains this in more detail.

In spite of these problems, the value of impact testing for modal analysis cannot be overstressed. It provides a quick means for troubleshooting vibration problems. For a great many structures an impact can suitably excite the structure such that excellent transfer function measurements can be made. The secret of its success rests with the user and his understanding of the physics of the situation and the basics of digital signal processing.

Step Relaxation Testing

Step relaxation is another form of transient testing which utilizes the same type of signal processing techniques as the impact test. In this method, an inextensible, light weight cable is attached to the structure and used to preload the structure to some acceptable force level. The structure "relaxes" with a force step when the cable is severed, and the transient response of the structure, as well as the transient force input, are recorded.

Although this type of excitation is not nearly as convenient to use as the impulse method, it is capable of putting a great deal more energy into the structure in the low frequency range. It is also adaptable to structures which are too fragile or too heavy to be tested with the hand-held hammer described earlier. Obviously, step relaxation testing will also require a more complicated test setup than the impulse method but the actual data acquisition time is the same.

Testing a Simple Mechanical System

A single-degree-of-freedom system was tested with each type of excitation method previously discussed. Besides measuring the linear characteristics of the system with each excitation type, gross non-linearity was simulated by clipping approximately one-third of the output signal. This condition simulates a "hard stop" in an otherwise unconstrained system. The intent of these tests was to show how certain forms of excitation can be used to measure the linear characteristics of a system with a large amount of distortion. This is extremely important to the engineer who is interested in identifying modal parameters.

Figure 9 illustrates the form of each type of stimulus and its power spectrum after fifteen ensemble averages. Notice that the input power spectrums for both the pure random and periodic random cases have more variance than the others. This is because each ensemble average consisted of a new and uncorrelated signal for these two stimuli. The pseudo random and swept sine (chirp) signals were controlled by the analyzer's digital-to-analog converter and each ensemble average was in fact the same signal, thus resulting in zero variance. In this test, the transient signal was also controlled by the DAC to obtain record-to-record repeatability and resulting zero variance. In all cases, the notching in the power spectrums is due to the impedance mismatch between the structure and the shaker. A final interesting note is that all spectrums except the swept sine are flat out to the cut-off frequency. The roll-off of the swept sine spectrum is due to the logarithmic sweep rate. Thus, the spectrum has reduced energy density as the frequency is increased.

Recall that in Part I we discussed the use of the coherence function to assess the quality of the transfer function measurement. In Figure 10, the results obtained from testing the single-degree-of-freedom system with and without distortion are shown. In Figures 10A and 10B the cases for pure random excitation, notice that the coherence is noticeably different from unity in the vicinity of the resonance. This is due to the non-periodicity of the signals and the fact that Hanning windowing was used to reduce what would have otherwise been even more severe leakage. The leakage effect is much more sensitive here, due to the sharpness of the resonance, i.e., the rate of change of the function. Although the effect is certainly present throughout the rest of the band, the relatively small changes in response level between data points away from the resonance will obviously tend to minimize the leakage from adjacent channels. Although any number of different windowing functions could have been used, the phenomenon would still exist.

Figures 10C-10J show the results of testing the system with the other excitation forms. In all figures showing the distorted case, the best fit of a linear model to the measured data is also shown. The coherence is almost exactly unity for the linear cases shown in Figures 10C, E, G and I. This is because all are ideally leakage-free measurements because they are periodic in the analyzer's measurement window. For the cases with distortion, the latter three show very good coherence even though the system output was highly distorted. This apparently good value of coherence is due to the nature of the zero-variance periodic signals used as stimuli. In cases 10B and 10D, the measurements are truly random from average to average and the coherence is more indicative of the

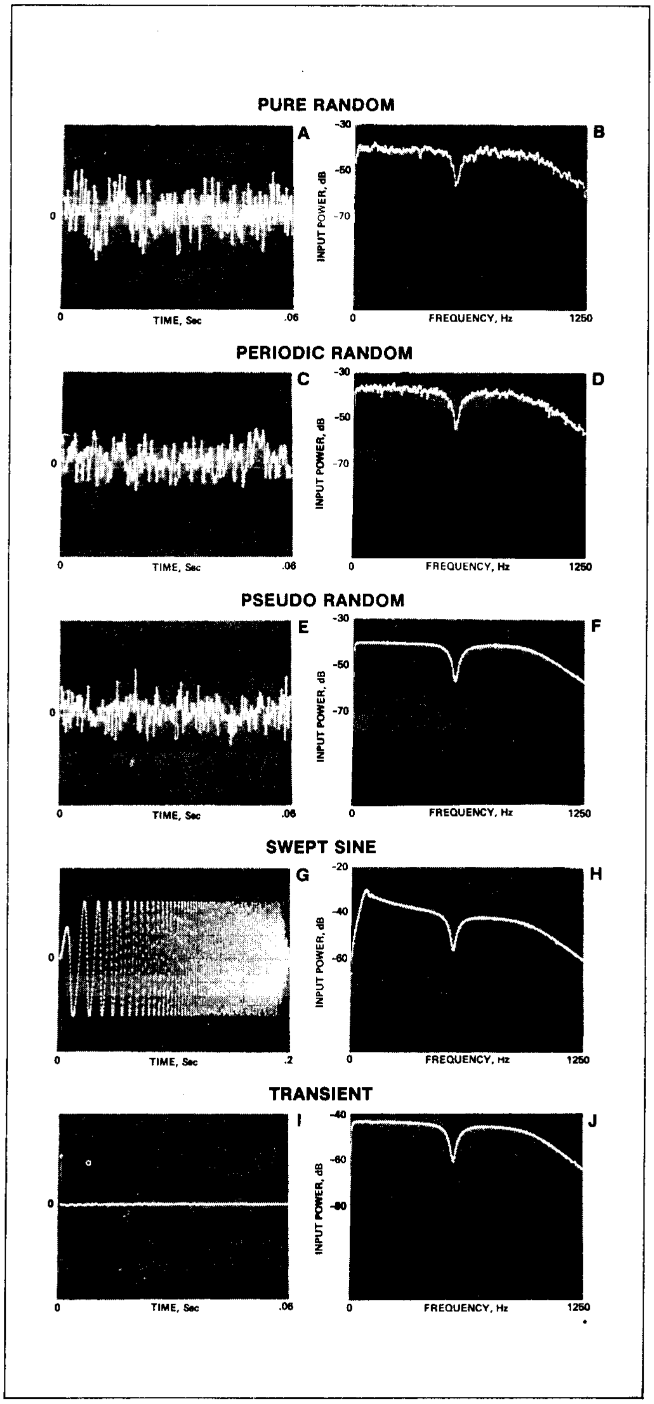

Figure 9—Different excitation types and their power spectrums. Each type was used to test a single-degree-of-freedom system. Fifteen ensemble averages were used.

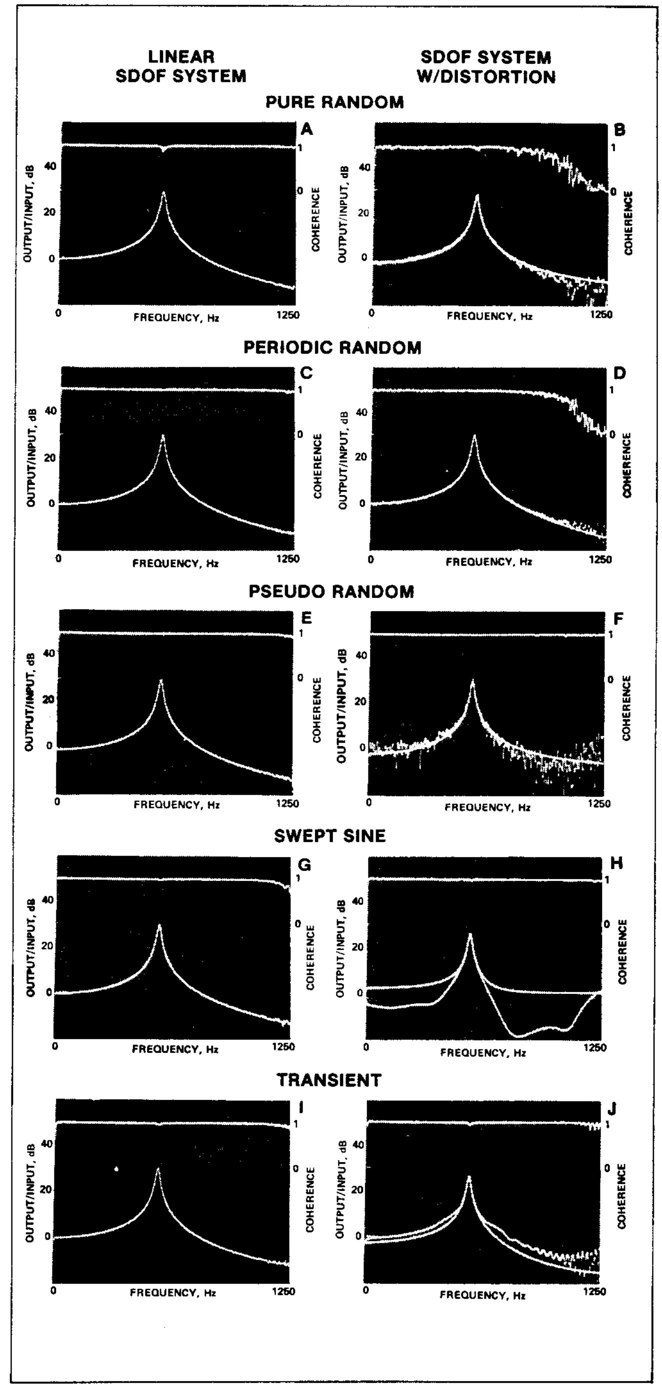

Figure 10—Comparison of different excitation types for testing the same single-degree-of-freedom system with and without distortion.

quality of the measurement. The low coherence values at the higher frequencies are primarily a result of the poor signal energy available. The conclusion is that the coherence function can be misleading if one does not understand the measurement situation.

Even though the system was highly distorted, it is apparent that the pure random and periodic random stimuli provided the best means for transfer function measurements, as seen in Figures 10B and 10D. Again, this is due to their ability to effectively use ensemble averaging to remove the distortion components from the measurement. The distortion cannot be removed using the other types of periodic stimuli and this is evident in Figures 10F, H and J. The results obtained from fitting a linar model to the measured data are given in Table I.

In all cases where the linear motion was measured, each type of excitation gave excellent results, as indeed they should. The one item worthy of mention is the estimate of damping with the pure random result. In this case, the value is about 7% higher than the correct

Table I – Linear model comparison of single-degree-of-freedom system resonance frequency and damping measurements using various excitation and analysis techniques

Test Condition	Frequency (Hz)	Damping Coefficient (rad/sec)	Magnitude	Phase (deg)	Relative Error
Pure Random…………………..	549.44	56.83	3429.12	0.5	23.1
Pure Random w/Distortion……	550.10	56.22	2963.28	357.1	21.7
Periodic Random………………	549.46	52.76	3442.18	0.6	3.6
Periodic Random w/Distortion..	549.50	53.44	3272.00	0.4	4.2
Pseudo Random……………….	549.55	52.76	3450.54	0.6	1.8
Pseudo Random w/Distortion…	550.09	51.75	2766.06	359.3	32.4
Swept Sine……………………..	549.49	53.24	3444.01	0.6	2.2
Swept Sine w/Distortion……….	549.77	53.76	2411.52	4.5	21.5
Transient………………………..	549.63	53.75	3453.26	0.7	5.7
Transient w/Distortion………….	549.68	53.13	2200.84	359.4	102.9
BSFA with Pure Random……..	549.44	53.12	3446.84	0.7	3.2

Table II – Principal characteristics of five excitation methods

Characteristics	Pure Random	Pseudo Random	Periodic Random	Impact	Swept Sine (Chirp)
Force level is easily controlled…………	Yes	Yes	Yes	No	Yes
Force spectrum can be easily shaped...	No	Yes	Yes	No	Yes
Peak-to-rms energy level………………	Good	Good	Good	Poor	Best
Requires a well-designed fixture and exciter system………………………..	Yes	Yes	Yes	No	Yes
Ensemble averaging can be applied to remove extraneous noise……………	Yes	Yes	Yes	Yes	Yes
Non-linearities and distortion effects can be removed by ensemble averaging……………………………..	Yes	No	Yes	Somewhat	No
Leakage Error…………………………..	Yes	No	No	Sometimes	No

value. This error is due to the windowing effect on the data. In this test, a Hanning window was used. However, any number of other windows could have been used and error would still be present. Further evidence of the Hanning effect on the data is shown by the error between the linear model and the measured data.

Of considerable importance is the data for the simulated distortion. The primary conclusion that can be drawn from these data is that the periodic random stimulus provides a good means for measuring the linear response of a linear system and is clearly superior to a pure random stimulus. It is also the best possible excitation for measuring the linear response of a system with distortion. Evidence of this is seen in the quality of the parameter estimates in Table I and the relative error (the error index between the ideal linear model and the measured data). The principal characteristics of each type of excitation are summarized in Table II.

Increasing Frequency Resolution

Certainly the single most important factor affecting the accuracy of modal parameters is the accuracy of the transfer function measurements. And, in general, frequency resolution is the most important parameter in the measurement process. In other words, it is simply not possible to extract the correct values of the modal parameters when there is inadequate information available to process. Modern curve fitting algorithms are highly dependent on adequate resolution in order to give correct parameter estimates, including mode shapes.

In this section we will introduce Band Selectable Fourier Analysis (BSFA), the so-called "zoom" transform. BSFA is a measurement technique in which the Fourier transform is performed over a frequency band whose lower and upper limits are independently selectable. This is in contrast to standard baseband Fourier analysis, which is always computed over a frequency range from zero frequency to some maximum frequency, $F_{\max}$. From a practical viewpoint, in many complex structures, modal density is so great, and modal coupling (or overlap) so strong, that increased frequency resolution over that obtainable with baseband techniques is an absolute necessity for achieving reliable results.

In the past, many digital Fourier Analyzers have been limited to baseband spectral analysis; that is, the frequency band under analysis always extends from dc to some maximum frequency. With the Fourier transform, the available number of discrete frequency lines (typically 1024 or 512) are equally spaced over the analysis band. This, in turn, causes the available frequency resolution to be, $\Delta f = F_{\max}/(N/2)$, where *N* is the Fourier transform block size, i.e., the number of samples describing the real-time function. There are $N/2$ complex (magnitude and phase) samples in the frequency domain. Thus, $F_{\max}$ and the block size, *N*. determine the maximum obtainable frequency resolution.

The problem with baseband Fourier analysis is that, to increase the frequency resolution for a given value of $F_{\max}$, the number of lines in the spectrum (i.e., the block size) must increase. There are two important reasons why this is an inefficient way to increase the frequency resolution:

1. As the block size increases, the processing time required to perform the Fourier transform increases.
2. Because of available computer memory sizes, the block size is limited to a relatively small number of samples (typically a maximum of 4096).

More recently, however, the implementation of BSFA has made it possible to perform Fourier analysis over a frequency band whose upper and lower frequency limits are independently selectable. BSFA provides this increased frequency resolution without increasing the number of spectral lines in the computer.

BSFA operates on incoming time domain data to the analyzer's analog-to-digital converter or time domain data that has previously been recorded on a digital mass storage device. BSFA digitally filters the time domain data and stores only the filtered data in memory. The filtered data corresponds to the frequency band of interest as specified by the user. The procedure is completed by performing a complex Fourier transform on the filtered data.

Of fundamental importance is the fact that the laws of nature and digital signal processing also apply to the BSFA situation. Since the frequency resolution is always equal to the reciprocal of the observation time of the measurement, $\Delta f = 1/T$, the digital filters must process T seconds of data to obtain a frequency resolution of $1/T$ in the analysis band. Whereas in baseband Fourier analysis the maximum resolution is always $\Delta f = F_{\max}/(N/2)$, the resolution with BSFA is $\Delta f = BW/(N/2)$ where BW is the independently selectable bandwidth of the BSFA measurement. Therefore, by restricting our attention to a narrow region of interest below $F_{\max}$ and concentrating the entire power of the Fourier transform in this interval, an increase in frequency resolution equal to $F_{\max}/BW$ can be obtained (Figure 11).

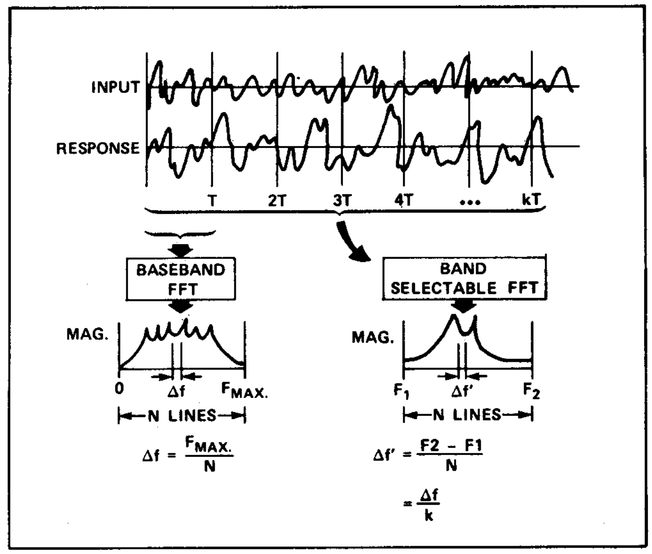

Figure11—Band Selectable Fourier Analysis™ versus baseband Fourier analysis. BSFA processes more data to obtain increased frequency resolution.

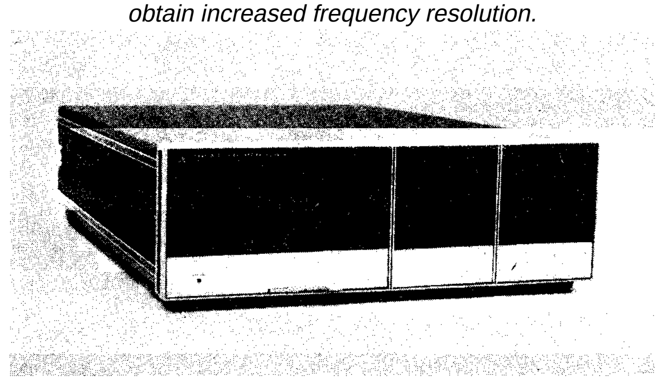

Figure 12—Hewlett-Packard's new 54470A Fourier Preprocessor gives the HP 5451B Fourier Analyzer greatly expanded capability for making Band Selectable Fourier Analysis measurements.

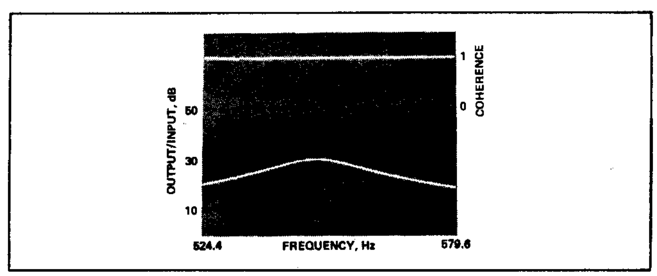

Figure 13—Pure random excitation and Band Selectable Fourier Analysis were used to test the single-degree-of-freedom system of Figure 10. The resolution is 18 times better than the baseband case shown in figure 10A. Note the improved coherence between the two sets of data, especially near the resonant frequency.

The other significant advantage of BSFA is its ability to increase the dynamic range of the measuremnt to 90 dB or more in many cases. The increased dynamic range of BSFA is a direct result of the extremely sharp roll-off and out-of-band rejection of the pre-processing digital filters and of the increased frequency resolution which reduces the effect of the white quantizing noise of the analyzer's analog-to-digital converter.[5] Certain types of BSFA filters can provide more than 90 dB of out-of-band rejection relative to a full scale in band spectral line, a characteristic which is not matched by more traditional analog range translators (see Figure 12).

The simple single-degree-of-freedom system which was tested with the various excitation types was also tested with BSFA using pure random excitation. We saw that in the baseband case, pure random was the least desirable signal because of the associated leakage and the resulting distortion of the transfer function waveform introduced by the Hanning window. By using BSFA, leakage is no longer an important source of error because of the great increase in the number of spectral lines used to describe the system. Figure 13 shows the coherence and transfer function between 524.6 Hz and 579.6 Hz with a resolution of 0.269 Hz, an increase of more than 18 over the baseband result. Note that the coherence is almost exactly unity, indicating the absence of any error due to leakage, and confirming the quality of the BSFA measurement. As shown in Table I, the use of BSFA eliminates the error caused by the leakage in the baseband measurement.

A Practical Problem. To illustrate the importance of BSFA, a mechanical structure was tested and modes in the area of 1225 Hz to 1525 Hz were to be investigated. Figure14 is a typical baseband ($\mathrm{dc} - F_{\max}$) transfer function measurement. It was taken with the following parameters:

Block size	1024
$F_{\max}$	5000 Hz
Filter cutoff	2500 Hz
Δf	9.765 Hz

Pure random noise was used to excite the structure through an electro-dynamic shaker.

The Inadequacy of the Baseband Measurement. Note that two modes are clearly visible between 1225 Hz and 1525 Hz. This same measurement is shown in rectangular or co/quad form in Figure I5. Again, by examining the quadrature response, the two modes are seen, However, there is also a slight inflection in the response between these two modes which indicates that yet a third mode may be present. But there is insufficient frequency resolution to adequately define the mode.

Returning to Figure 15, a partial display of the region between 1225 and 1525 Hz was made. The expanded quadrature display is shown in Figure 16. Realize that this represents no increase in frequency resolutions only an expansion of the plot. Clearly only two modes were found.

Accurate Measurements with BSFA. In order to accurately define the modes in this region, the structure was re-tested using Band Selectable Fourier Analysis (BSFA). All 512 lines of spectral resolution were placed in a band from 1225 to 1525 Hz, resulting in a resolution of 0.610 Hz instead of 9.76 Hz, as in the baseband

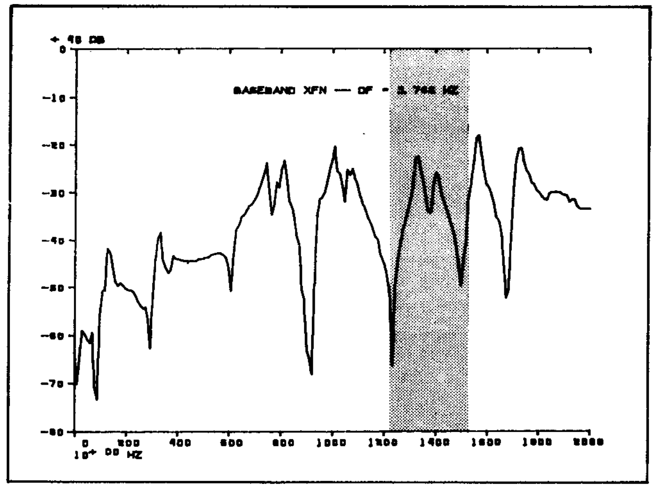

Figure 14—Baseband transfer function shows two modes at approximately 1320 Hz and 1400 Hz.

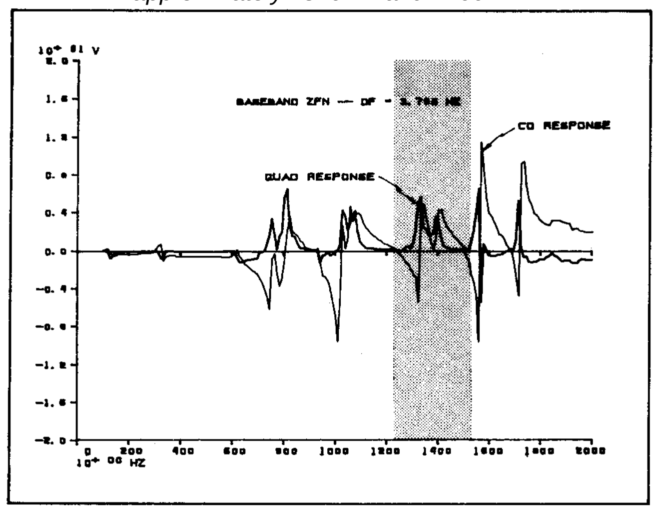

Figure 15—Baseband measurement in co/quad form shows two major modes and a slight inflection between the two which possibly indicates a third mode. However, there is not enough resolution in the measurement to be sure or to identify the mode.

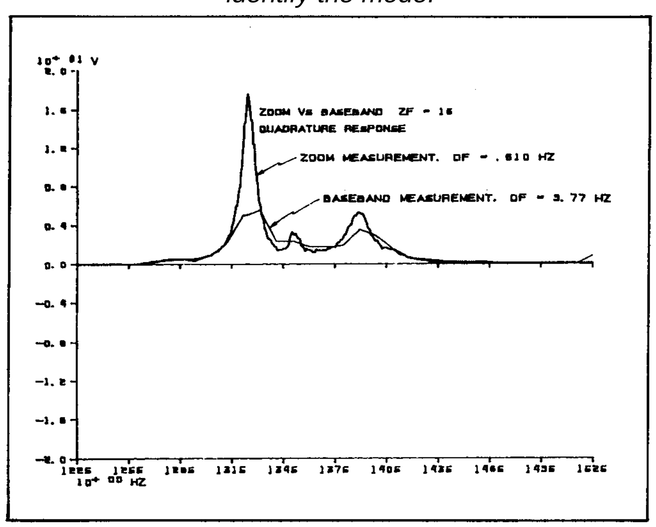

Figure 16—Comparison of quadrature response of the baseband and BSFA result. The BSFA measurement clearly shows the small third mode and the poor result of the baseband measurement for the other two modes.

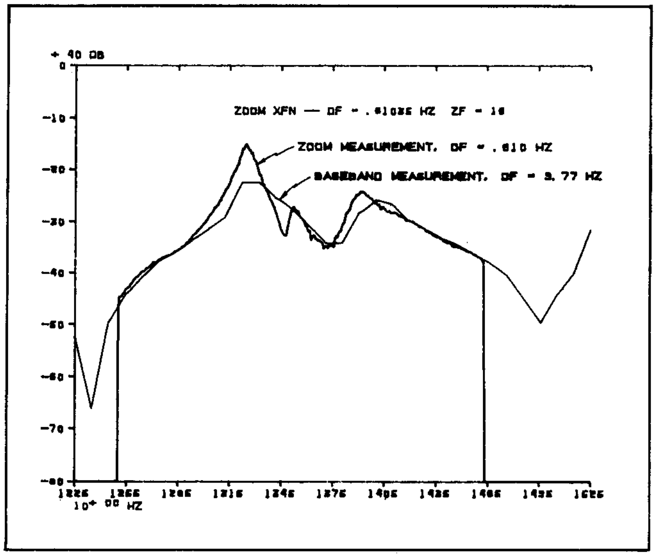

Figure 17—Comparison of the BSFA and baseband transfer functions between 1225 and 1525 Hz. In the BSFA result, three modes are clearly visible and well defined. The baseband data would have led to considerable error in estimates of frequency and damping.

measurement. The quadrature response attained with the BSFA is also shown in Figure 16 for comparative purposes. Note that three modes are now clearly visible. The small (third) mode of approximately 1350 Hz is now well defined, whereas it was not even apparent before. In addition, the magnitude of the first mode at 1320 Hz is seen to be at least three times greater in magnitude than the result indicated by the baseband measurement. The corresponding results in log form are shown in Figure 17. This BSFA result was obtained by using only a 16:1 resolution enhancement. Enhancements of more than 100:1 are possible with BSFA.

Implications of Frequency Resolution in Determining Modal Parameters and Mode Shapes. Referring again to Figure 15, we can clearly see the necessity of using adequate frequency resolution for making a particular measurement. In addition, it is important to understand how the baseband result would lead to an incorrect answer in terms of modal parameters and mode shape.

1. **Modal Parameters.** If the baseband result is compared to the BSFA result for the 1320 Hz mode it is obvious that the baseband result would indicate that the mode is much more highly damped than it actually is. The second small mode (1350 Hz) would not even be found, and the 1400 Hz mode would also have the wrong damping. Close inspection also shows that the estimate of the resonance frequency for the 1320 Hz mode would have significant error.

2. **Mode Shape.** Any technique for estimating the mode shape coefficients (e.g., quadrature response, circle fitting, differencing, least squares, etc.) would clearly be in error since it is apparent that the BSFA result shows a quadrature response at least three times greater than the baseband result.

Although the proceeding example presented a case where the use of BSFA was a necessity, it is very easy for the engineer to be misled into believing he has made a measurement of adequate resolution when in fact he has not. The following concluding example illustrates this point and presents the estimates of the modal parameters for each case.

A disc brake rotor was tested using an electro-dynamic shaker and pure random noise as a stimulus. A load cell was used to measure the input force and an accelerometer mounted near the driving point was used to measure the response. The baseband measurement had a resolution of 9.76 Hz. As can be seen in Figure 18A, the two major modes at about 1360 Hz and 1500 Hz appear to be well defined. An expanded display (no increased resolution) from 1275 Hz to 1625 Hz clearly shows the two large modes and a much smaller mode at about 1580 Hz.

The rotor was re-tested using BSFA and the two sets of data are compared in Figure 18. This data clearly shows the value of BSFA. The BSFA data provides increased definition of the modal resonances, as can be seen by comparing the baseband and BSFA results. The validity of each result is reflected in the respective coherence functions. The baseband transfer function contains inaccuracies due to the Hanning effect, as well as inadequate resolution. The coherence for the BSFA measurement is unity in the vicinity of all three modal resonances, indicating the quality of the transfer function measurement. Further proof of the increased modal definition is shown in the BSFA Nyquist plot (co versus quad). Here, all three modes are clearly discernible and form almost perfect circles, indicating an excellent measurement, almost totally free of distortion. In the baseband result, only three or four data points were available in the vicinity of each resonance, whereas in the BSFA data many more points are used.

The modal parameters for all three modes were identified from the baseband and BSFA data and the results are shown in Table III. Comparison of results emphasizes the need for BSFA when accurate modal parameters are desired.

In summary, no parameter identification techniques are capable of accurately identifying modal parameters or mode shapes when the frequency resolution of the measurement is not adequate.

Summary

We have seen that frequency response functions can be used for identifying the modes of vibration of an elastic structure and that the accurate measurement of the frequency response functions is the most important factor affecting the estimates of the modal parameters.

Pure random, pseudo random, periodic random, swept sine, and transient techniques for baseband Fourier analysis were discussed. All types of stimuli, except for pure random, gave excellent results when used for testing a linear system. The pure random result contained some error because its non-periodicity in the measurement window required that Hanning be used on

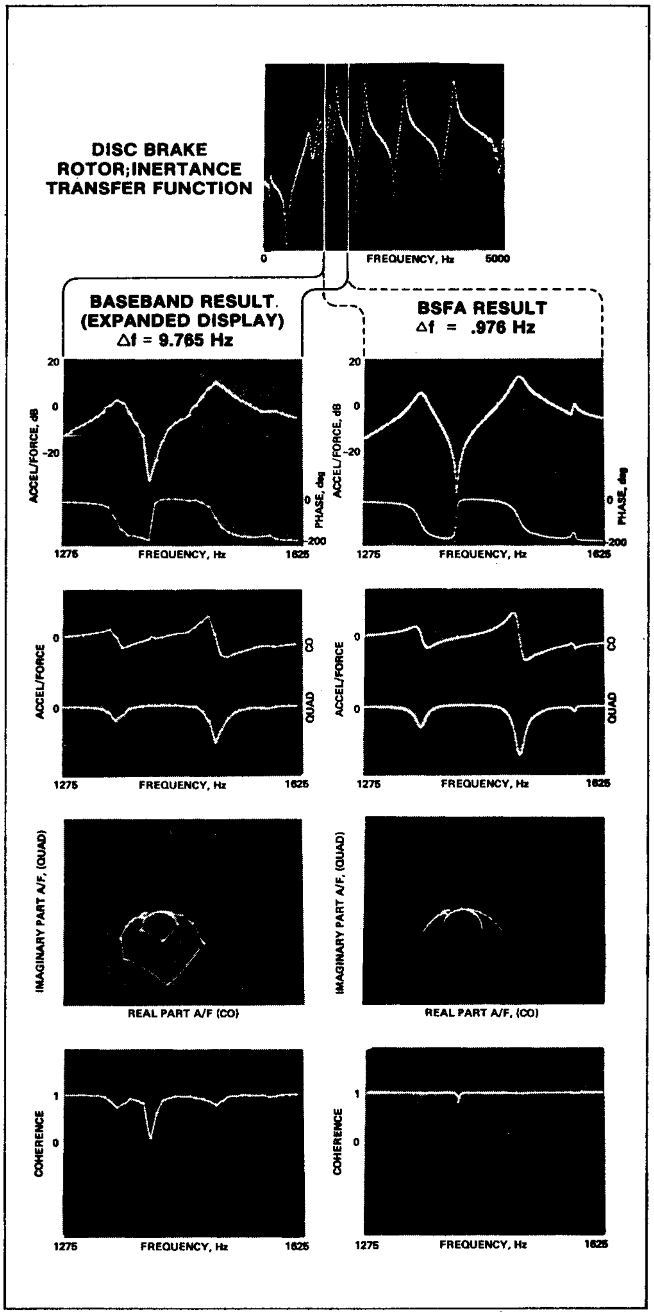

Figure 18—Baseband and BSFA inertance transfer functions from a disc brake rotor. The BSFA result gives a better estimate of the transfer function by using more data points. This results in a better estimate of the modal parameters as shown in Table III. Note the improved coherence and the clear definition of all three modes in the Nyquist display (co versus quad).

the input and response waveforms, resulting in some distortion of the transfer function.

For systems with distortion, periodic random offers significant advantages over the other types of stimuli. It is best able to measure the linear response of distorted systems. This means that modal parameters extracted from transfer functions measured with periodic random will be more accurate. None of the techniques discussed are capable of compensating for inadequate frequency resolution.

Table III – Comparison of modal parameter test results

Baseband Results, Δf = 9.765 Hz

Mode	Frequency, Hz	Damping, %	Amplitude	Phase
1	1359.99	0.775	193.51	350.3
2	1503.92	0.763	483.30	11.1
3	1584.33	0.273	9.49	336.1

BSFA Results, Δf = 0.976 Hz

Mode	Frequency, Hz	Damping, %	Amplitude	Phase
1	1359.13	0.669	211.99	352.7
2	1502.65	0.652	509.52	9.4
3	1583.50	0.131	11.65	340.8

Error, %, Versus Baseband

Mode	Frequency, Hz	Damping, %	Amplitude
1	0	16%	8%
2	0	17%	5%
3	0	108%	19%

Band Selectable Fourier Analysis was introduced as a means for arbitrarily increasing the frequency resolution of the frequency response measurement by more than 100 times over standard baseband measurements. BSFA's increased resolution provides the best possible means for making measurements for the identification of modal parameters and, in a great number of practical problems, is the only feasible approach.

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